Radio Link Power Density and Antenna Factor

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Abstract—Generally antenna characteristics have been defined always in the transmitting case. In free space radio link identical antennas characteristics show that they have exactly the same gain, area, and factor. Contrary to current usage and practices transmitting and receiving antenna characteristics *are not* the same, since the surface of the earth acts as a reflecting plane. This investigation was conducted to show the difference in gain, area, and factor of two identical antennas in the transmitting and receiving role, through calculations, simulation, and measurements necessary for electromagnetic compatibility.

Index Terms—Antenna radiation pattern, dipole antenna, gain.

I. INTRODUCTION

NTENNA factor (af) is defined as the relationship between the incident electric field strength E_i on a receiving antenna and the voltage V_R developed on its receiving load R_L . In a radio link over perfect ground as shown in Fig. 1 and according to the equivalent Thevenin circuit shown in Fig. 2 is represented by:

$$af = \frac{E_i}{V_R} \quad [1/m] \tag{1}$$

In this radio link, transmission power loss a_w (Friis equation) is expressed as [3], [6]:

$$a_w = \frac{W_R}{W_T} = \frac{g_T A_{eR}}{4\pi (r')^2}$$
(2)

where g_T is the natural numerical transmitting antenna gain. A_{eR} is the natural receiving antenna effective area $[m^2]$.

These parameters are defining the behavior of both antennas operating in a different way in the transmitting and receiving role. Their definitions are [4], [5]:

$$g_T = \frac{4\pi U_M}{W_T} \tag{3}$$

$$A_{eR} = \frac{W_R}{P_i} \quad [m^2] \tag{4}$$

where U_M is the maximum wave power intensity

 $U_M = P_i(r')^2$. W_T is the transmitted power [W].

 W_R is the receiving antenna power [W].

 P_i is the incoming wave power density [W/m²].

The transmitting antenna gain is related to an isotropic radiator whose power is radiated uniformly in any surrounding space

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Fig. 1. Radiolink geometry.

directions and whose effective area is found to be [4], [5]

$$A_{e0} = \frac{\lambda^2}{4\pi} \quad [\mathrm{m}^2]. \tag{5}$$

Taking into account the isotropic radiator properties Friis auxiliary equations can be achieved, as [4]:

$$a_w = g_T g_R a_{\rm FS} \tag{6}$$

$$a_w = \frac{A_{eT} A_{eR}}{\lambda^2 (r')^2} \tag{7}$$

where g_R is the numerical parameter obtained from the receiving antenna effective area A_{eR} and is called the receiving antenna gain g_R , or [2], [4]:

$$g_R = \frac{4\pi A_{eR}}{\lambda^2} \tag{8}$$

 A_{eT} is the parameter obtained from the transmitting antenna gain g_T and is called the transmitting antenna effective area A_{eT} , or [4]:

$$A_{eT} = \frac{\lambda^2 g_T}{4 \pi} \quad [\text{m}^2] \tag{9}$$

 $a_{\rm FS}$ is the free space nondissipative attenuation due to the electromagnetic wave spreading into space, thus:

$$a_{\rm FS} = \left(\frac{\lambda}{4\,\pi\,(r')}\right)^2\tag{10}$$

Using the Friis equation and the receiving antenna equivalent Thevenin circuit for a perfectly matched and resonant antenna according to Fig. 2 the receiving antenna factor can be obtained by calculations or measurements.

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Fig. 2. Receiving antenna equivalent Thevenin circuit.

II. ANTENNA FACTOR

A. Calculations

Two parameters are necessary to obtain the receiving antenna factor value over perfect ground. These are the incoming wave power density P_i or the far field electric intensity E_i and the voltage V_R developed on the receiving antenna resistive load R_L . The power density P_i is obtained, as:

$$P_i = \frac{W_T g_T}{4\pi (r')^2} \quad [W/m^2]$$
(11)

and the field strength E_i , by:

$$E_i = (P_i Z_{00})^{(0.5)} \quad [V/m] \tag{12}$$

where Z_{00} is the free space intrinsic impedance, or:

$$Z_{00} = 120 \,\pi \simeq 377 \,[\Omega] \tag{13}$$

The open-circuit induced voltage on the receiving antenna is found to be [7]:

$$V_i = L_e E_i \quad [V] \tag{14}$$

where L_e is the receiving antenna effective length. For a halfwave dipole antenna its value is:

$$L_e = \frac{\lambda}{\pi} \quad [m] \tag{15}$$

This way the current and voltage on the resistive load for $R_a = R_L, X_a = 0$ is obtained, as:

$$I_R = \frac{V_i}{2R_a} = \frac{V_i}{2R_L} \quad [A] \tag{16}$$

$$V_R = I_R R_L \quad [V] \tag{17}$$

The receiving antenna factor af is immediately obtained, thus:

af
$$= \frac{E_i}{V_R} = \frac{2 E_i}{V_i} = \frac{2}{L_e}$$
 [1/m] (18)

in dB/m, as:

$$AF = 20 \cdot \log af \quad [dB/m] \tag{19}$$

As an example the antenna factor for a radio link between two half-wave dipole antennas for horizontal and vertical



Fig. 3. Receiving antenna factor at r = 30 m for horizontal and vertical polarization as a function of the receiving antenna height H_R .

polarization has been calculated as a function of the receiving antenna height H_R and frequency between 30 and 300 MHz and it is shown in Fig. 3. Also, the receiving antenna effective or capture area and its gain can be obtained from:

$$A_{eR} = \frac{W_R}{P_i} = \frac{I_R^2 R_L}{P_i} = \frac{Z_{00} L_{eR}^2}{4 R_L} \quad [m^2]$$
(20)

$$g_R = \frac{4\pi A_{eR}}{\lambda^2} \tag{21}$$

in dBi:

$$G_R = 10 \cdot \log g_R \quad [dBi] \tag{22}$$

The receiving antenna factor taking into account the effective area A_{eR} can also be calculated by:

$$\operatorname{af}_{R} = \frac{\pi}{\lambda} \left(\frac{480}{R_{aR} g_{R}} \right)^{(0.5)} \quad [1/\mathrm{m}] \tag{23}$$

in dB and as function of frequency in MHz is found to be:

$$AF_{R73} = -31.42 + 20 \cdot \log f_{(MHz)} - 10 \cdot \log g_R \quad [dB/m]$$
(24)



Fig. 4. Power measurement set up.

This same equation can be used to calculate the transmitting antenna factor, or:

$$af_T = \frac{\pi}{\lambda} \left(\frac{480}{R_{aT} g_T} \right)^{(0.5)} [1/m]$$
(25)

$$AF_{T73} = -31.42 + 20 \cdot \log f_{(MHz)} - 10 \cdot \log g_T \quad [dB/m]$$
(26)

where R_{aR} and R_{aT} are the radiation resistance of the receiving and transmitting antennas, respectively. It is important to notice the continuous transmitting antenna factor variation according to the receiving antenna height, because the receiving antenna factor is almost constant with its height [7]. This is due to the transmitting antenna radiation pattern gain as a function of the elevation angle α_L . The transmitting antenna effective length L_{eT} can be obtained knowing the effective area A_{eT} , or:

$$L_{eT} = \left(\frac{4 R_{aT} A_{eT}}{Z_{00}}\right)^{(0.5)} \quad [m]$$
 (27)

B. Measurements

Two parameters are necessary to determine the receiving antenna factor over perfect ground. They are the transmitting antenna power W_T and the receiving antenna power W_R . Transmitting antenna input power W_T and receiving antenna power W_R can be measured by means of two power meters shown in Fig. 4. Transmitted power is measured into the transmitting antenna terminals by means of a 3 dB coaxial power splitter and a power meter. In order to check whether the transmitted power into space is constant its value can be continuously monitored. The transmission line loss connecting the generator and the antenna is eliminated, independent of the transmission line length. The received power can be measured by means of a spectrum analyzer or a sensitive power meter whose sensor could be installed directly in the receiving antenna terminals avoiding the transmission line losses and improving the measurements. The



Fig. 5. Radio link elevation angle at maximum received power W_R .

receiving power W_R permits, according to the Fig. 2, determining the equivalent Thevenin current I_R and the voltage V_R over the load resistance R_L , thus:

$$I_R = \left(\frac{W_R}{R_L}\right)^{(0.5)} \quad [A] \tag{28}$$

$$V_R = (W_R R_L)^{(0.5)}$$
 [V]. (29)

The open-circuit induced voltage is found to be:

$$V_i = 2 I_R R_L = 2 I_R R_{aR} \quad [V]. \tag{30}$$

The incoming wave field strength is obtained through the receiving antenna effective length, or:

$$E_i = \frac{V_i}{L_e} \quad [V/m]. \tag{31}$$

The receiving antenna factor af_R (1), effective area A_{eR} and its gain g_R can be obtained as previously in A. Using the gain auxiliary Friis equation the transmitting antenna gain g_T and effective area A_{eT} can be calculated, as:

$$G_T = K(\mathbf{dB}) - G_R(\mathbf{dBi}) \quad [\mathbf{dBi}] \tag{32}$$

$$u_T = 10^{(G_T/10)} \tag{33}$$

where

$$K(\mathbf{dB}) = A_w - A_{\mathrm{FS}} \quad [\mathbf{dB}] \tag{34}$$

$$A_{\perp} = 10 \cdot \log a_{\perp} \quad [dB] \tag{35}$$

$$A_{\rm FS} = 10 \cdot \log a_{\rm FS} \ [\rm dB] \tag{36}$$

and the transmitting antenna effective area AeT is:

$$A_{eT} = \frac{\lambda^2 g_T}{4 \pi} \quad [\mathrm{m}^2]. \tag{37}$$

III. EXAMPLE

Calculation have been performed in a radio link whose distance r = 10 [m] over perfect ground with two identical horizontally polarized half-wave dipole antennas according to Fig. 5 and using WIPL-D Software [8].

Antenna parameters are calculated when the incoming power density P_i on the receiving antenna corresponds to the



Fig. 6. Half-wave dipole transmitting antenna radiation pattern gain as a function of elevation angle at f = 300 MHz.

transmitting antenna maximum gain g_T as shown in Fig. 6. Fig. 6 shows the gain radiation pattern as a function of the elevation angle α .

Data:

Frequency f = 300 MHz. Transmitting power $W_T = 1$ [W]. Transmitting antenna height $H_R = 2$ [m].

Results:

At a maximum power density P_i on the receiving antenna. Rx antenna height $H_R = 4.1$ [m].

Radio link elevation angle $\alpha_L = 22.3[\circ]$.

Power density flowing distance r' = 10.81 [m] (Between Tx antenna array phase center and Rx antenna center).

Rx antenna effective area $A_{eR} = 0.13 \, [\text{m}^2]$.

Tx antenna effective area $A_{eT} = 0.52 \,[\text{m}^2]$.

Rx antenna gain $G_R = 2.16$ [dBi].

Tx antenna gain $G_T = 8.16 \,[\text{dBi}]$ at $\alpha_L = 22.3 \,[\circ]$.

Rx antenna factor $AF_{R(73)} = 15.96 \,[dB/m]$.

Tx antenna factor $AF_{T(73)} = 9.96 \,[dB/m]$.

Rx antenna effective length $L_{eR} = 0.3183$ [m].

Tx antenna effective length $L_{eT} = 0.6366$ [m].

Total real power density $P_i = 4.45 \cdot 10^{(-3)} \, [\text{W/m}^2]$.

y real power density component $P_{iy} = 4.00 \cdot 10^{(-3)} [\text{W/m}^2]$. z real power density component $P_{iz} = 1.61 \cdot 10^{(-3)} [\text{W/m}^2]$.

Field strength $E_i = 1.3$ [V/m].

Rx power over $R_L = 73 \ [\Omega]$

Received power $W_R = 5.85 \cdot 10^{(-4)}$ [W].

Real power density is seen clearly flowing in the radio link elevation angle $\alpha_L = 22$ [\circ], as shown by Figs. 5 and 7. Fig. 8 shows the ratio in dB between the real and imaginary power density as function of the receiving antenna height over ground. Note the sharp peak where the maximum power density is located and it corresponds at each lobe maximum. The ratio between the transmitting and receiving antenna parameters to be mention as effective lengths L_e , effective areas A_e , gains g, and



Fig. 7. Radio link power density components at a receiving antenna height $H_R = 4.1$ m and f = 300 MHz.



Fig. 8. Radio link power density relationship as a function of receiving antenna height H_R .

antenna factors af shows a 6 dB value in favor of the transmitting antenna for this maximum power density elevation angle. This effect shows a clear different behavior of the transmitting and receiving antenna operation over perfect ground. Only in free space both identical antennas have the same parameter values.

"Therefore, one must conclude that their performance over perfect ground is different."

In this example, near the minimum power density in the transmitting antenna radiation pattern, a variation in the free space intrinsic impedance is shown, whose minimum value corresponds exactly to the receiving antenna height $H_R = 2.6$ (m). At the same time, a minimum ratio of the real to imaginary power density component is obtained. The contrary is obtained at the transmitting antenna radiation pattern maximum located at $H_R = 4.1$ (m). In this case, the free space intrinsic impedance is practically the theoretical value (120π) and with a smooth local variation as a function of the receiving antenna height. This is represented in Fig. 9. The Transmitting Antenna Radiation Pattern is shown in Fig. 6, for a frequency f = 300 (MHz).

The calculated receiving antenna factor AF_{R73} as a function of the antenna height can be seen in Fig. 10 with reference to $R_L = 73(\Omega)$. It is interesting to observe practically the same result in dB/m for any height as a relationship of the effective



Fig. 9. Free space intrinsic impedance as a function of receiving antenna heightr H_R .



Fig. 10. Calculated receiving antenna factor AF_{R73} as a function of receiving antenna height H_R (dB/m) at f = 300 MHz.

electric field strength and the voltage V_R over the resistive antenna load R_L in Fig. 10.

The average value of the measured antenna factor is $AF_{R73} = 15.98$ (dB/m). This can be compared to the Rohde and Schwarz half-wave dipole antenna manual giving a value of $AF_{R73} = 15.96$ (dB/m) at a frequency f = 300 (MHz).

IV. HALF-WAVE DIPOLE ANTENNA MEASUREMENTS IN A SEMI-ANECHOIC CHAMBER

In order to verify the gain and antenna factor of a half-wave dipole antenna over ground, measurements were performed in an semi-anechoic chamber with metallic floor. The transmitting dipole antenna was installed with a fixed height H_T of one wavelength at a frequency of 150 MHz ($\lambda = 2$ m). The receiving half-wave dipole antenna was installed over a suitable dielectric support permitting the height variation from one to four meters. The distance between antennas was r = 10 m.

Fig. 11 shows the receiving antenna measurement setup. Both antennas are HZ-12 Rohde and Schwarz measuring antennas used by INTI of Argentina for electromagnetic compatibility certifications. The received power W_{R1} was



Fig. 11. Half-wave dipole antenna measurements setup (INTI Buenos Aires, Argentina).

TABLE I $A_W (DB) = W_r (DBW) - W_t (DBW)$

H_R (m)	α (°)	<i>r</i> ₁ (m)	W_{R1} (dBW)	A_W (dB)	$A_{\rm FS}$ (dB)	P_i (W/m ²)
1.00 1.25 1.50 1.75 2.00 2.50 3.00 3.50 4.00	5.70 7.10 8.50 9.90 11.3 14.0 16.7 19.3 21.8	10.05 10.08 10.11 10.15 10.20 10.30 10.44 10.59 10.77	-71.22 -68.82 -68.78 -68.58 -67.67 -67.80 -67.92 -68.70 -69.63	$\begin{array}{r} -30.34\\ -27.94\\ -27.90\\ -27.70\\ -26.79\\ -26.92\\ -27.04\\ -27.82\\ -28.75\end{array}$	$\begin{array}{r} -36.01 \\ -36.03 \\ -36.06 \\ -36.09 \\ -36.14 \\ -36.22 \\ -36.34 \\ -36.46 \\ -36.61 \end{array}$	$\begin{array}{c} 1.44 \times 10^{-7} \\ 2.51 \times 10^{-7} \\ 2.53 \times 10^{-7} \\ 2.65 \times 10^{-7} \\ 3.27 \times 10^{-7} \\ 3.17 \times 10^{-7} \\ 3.08 \times 10^{-7} \\ 2.58 \times 10^{-7} \\ 2.08 \times 10^{-7} \end{array}$

TABLE II $K = A_W - A_{FS}(DB)$

<i>H</i> _{<i>R</i>} (m)	A_{eR} (m ²)	G_R (dBi)	К (dB)	G _T (dBi)
1.00	0.5233	2.16	5.67	3.51
1.25	0.5221	2.15	8.09	5.94
1.50	0.5218	2.15	8.16	6.01
1.75	0.5246	2.17	8.39	6.22
2.00	0.5232	2.16	9.35	7.19
2.50	0.5234	2.16	9.30	7.14
3.00	0.5220	2.15	9.30	7.15
3.50	0.5202	2.13	8.64	6.51
4.00	0.5231	2.16	7.86	5.70

determined measuring the power W_R in a spectrum analyzer. This can be seen in Tables I–III as a function of the receiving antenna height. Taking into account the effective length (L_e = 0.6366 m) and the antenna impedance ($R_A = 73 \Omega$) the receiving power density P_i was calculated.

From these measurements, the receiving antenna has a gain of 2.16 (dBi) almost constant as a function of height. The transmitting antenna gain obtained corresponds to the radiation pattern at a height of one wavelength. The transmitting antenna gain was calculated theoretically by means of WIPL-D Software.

Both gains have been obtained here knowing only the measured transmitted and received power.

 H_R E_i V_R AF AF_{73} (V/m)(V) (dB/m)(m) (-)1.00 7.38×10^-3 2.35×10^{-3} 3.14 9.94 1 25 9.73×10^{-3} 3.10×10^{-3} 3 14 9 94 1.50 9.77×10^-3 3.18×10^{-3} 9.94 3.14 1.75 9.94 9.99×10^{-3} 3.14 3.18×10^{-3} 2.00 1.11×10^{-2} 3.53×10^{-3} 3.14 9.94 9.91 2.50 $1.09 imes 10^{-2}$ 3.48×10^{-3} 3.13 3.00 1.08×10^-2 3.43×10^{-3} 3.15 9.97 3.50 9.85×10^{-3} 3.13×10^{-3} 3.15 9.97 8.86×10^-3 4.00 2.82×10^{-3} 3.14 9.94

TABLE III Antenna Factor (dB/m)



Fig. 12. Calculated and measured gain of two half-wave dipole antennas in a semi-anechoic chamber.



Fig. 13. Half-wave dipole transmitting antenna gain calculated as a function of elevation angle α at f = 150 MHz.

This can be seen in Fig. 12. Here the Friis equation is valid over ground (see Fig. 12). The calculated transmitting antenna gain as a function of the elevation angle α_P is shown in Fig. 13. The measured gain values are very close to the theoretically calculated values obtained by Software. Antenna factors can be seen in Tables III, where the obtained values by measurements are practically those indicated by Rohde and Schwarz for f =



Fig. 14. Picture of the semi-anechoic chamber, INTI Buenos Aires.

150 MHz (AF₇₃ = 9.92 (dB/m)). INTI Buenos Aires, picture of the semi-anechoic chamber can be seen in Fig. 14.

V. SUMMARY—SIMPLE ANTENNA GAIN, EFFECTIVE AREAS AND EFFECTIVE LENGTHS

Half-wave dipole over perfect ground:

1) Transmitting $(R_a \cong 73 \Omega, X_a = 0)$: Max. Effective length: $\frac{L_{eT}}{\lambda} = 0.63 \ (-3.95 \text{ dB})$. Max. Effective area: $\frac{A_{eT}}{\lambda^2} = 0.52 \ (-2.84 \text{ dB})$. Max. Transmitting gain: $g_T = 6.55 \ (8.16 \text{ dBi})$. Max. antenna factor: $AF_{T73} = -39.58 + 20 \cdot \log f_{(MHz)}$. Max. is obtained at the radiation pattern gain maxi-

Max. is obtained at the radiation pattern gain maximum.

2) Receiving $(R_a \cong 73 \Omega, X_a = 0)$:

Effective length: $\frac{L_{eR}}{\lambda} = 0.318 \ (-9.94 \ \text{dB}).$ Effective area: $\frac{A_{eR}}{\lambda^2} = 0.13 \ (-8.84 \ \text{dB}).$ Receiving gain: $g_R = 1.64 \ (2.16 \ \text{dB}i).$ Antenna factor: $AF_{R73} = -33.58 + 20 \cdot \log f_{(MHz)}.$

VI. CONCLUSION

Calculations, simulation, and measurement of radio links with antennas installed over a reflective surface, such as the earth surface, indicate that the gain of the receiving antenna is -6 dB relative the same identical antenna used in the transmitting mode. The results are valid for linear antennas, such as dipoles installed for vertical or horizontal polarization. This fact can also be verified for directional antennas. It can be observed that transmitting antenna installed over a reflecting surface behaves as an array of two identical radiating elements. This is an antenna and its image. The receiving antenna will receive energy from both sources; this energy being identical as if the reflective surface has infinite conductivity and perfectly smooth and the coefficient of reflectivity is one. The receiving antenna, on the other hand gets no added contribution from its image on its load impedance. The receiving antenna does not behave as an array of two elements equal to the transmitting antenna and for this reason they differ in area, gain, and factor. As shown, the receiving antenna operates as if it is in free space. For this reason its gain is practically constant as a function of height over ground as indicated by the results of different experiments.

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