Short Low and Medium Frequency Antenna Performance

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Abstract— **Lately, short antennas have attracted broadcast and communication community attention.**

This kind of antennas has been used since the 1920's.

Top-loaded monopoles are the logical antennas to be used in order to get a low profile antenna and a performance according to the broadcaster and communication needs.

In this paper, top-loaded monopoles have been studied exhaustively using the transmission line technique, obtaining improved expressions for the antenna radiation resistance taking into account the top-base current relationship and under different top-loading conditions.

This idea of using an equivalent transmission line technique has been used since the 1920's in order to obtain the antenna input reactance.

Using this old idea, the novel approach here permits to obtain the near and far field expressions from the current distribution on the antenna structure. Near field calculation is used to determine the surface current density on the ground plane.

From the artificial and natural ground plane surface current density, the power dissipation is calculated and the ground plane equivalent loss resistance is obtained.

In all cases, as a first approximation, a half-wavelength ground plane radius has been used, because this is the maximum distance covered by the ground surface current under the antenna, closing the antenna electric circuit. Beyond this distance, the ground currents do not return to the antenna generator and are taken into account in the surface wave propagation calculations. The half-wavelength ground plane surface is partially occupied by the metallic radial ground system and the remainder by the natural soil.

Artificial ground plane behavior is paramount in obtaining the best performance of a short antenna. This kind of antennas could perform very close to a standard quarter-wave monopole if they work with optimum dimensions. For these reasons, a short antenna and the corresponding artificial ground plane have been analyzed modifying the number of radials and their lengths, in order to achieve an optimum performance or to obtain maximum field strength on several soil conditions of the earth surface.

A very simple and efficient antenna could be obtained, giving to the broadcast and communication community a product that could fulfill the required performance to radiate a high quality AM or digital transmission on MF band, and good speech quality on LF band.

Index Terms— **Antennas, Antenna Theory, LF Antennas, LF Broadcast Antennas, LF Broadcast Transmitting Antennas, LF Top-Loaded Antennas, LF AM Broadcast Antennas, LF Monopole Antennas, MF Antennas, MF Broadcast Antennas, MF Broadcast Transmitting Antennas, MF Top-Loaded Antennas, LF Short Transmitting Antennas, MF Short Transmitting Antennas, MF AM Broadcast Antennas, MF Monopole Antennas, Artificial Ground Plane Antenna, Artificial Ground Plane Metallic Radials, Artificial Ground Plane Impedance, Antenna Input Impedance, Antenna Efficiency, Antenna Gain, Antenna Performance, Antenna Bandwidth, Antenna Wiring.**

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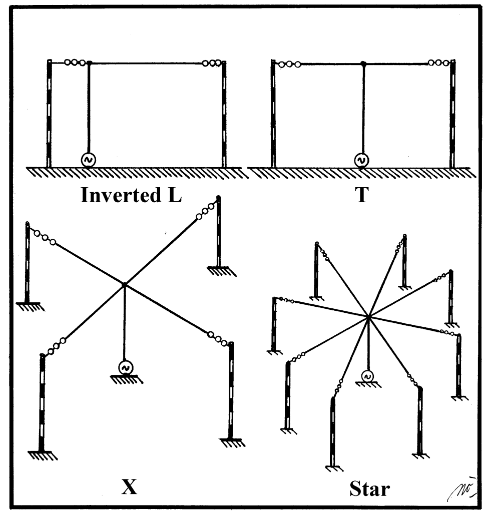

Fig. 1. Top-loaded antennas.

I. Introduction

In a previous paper [1] several problems concerning short antennas have been pointed out.

Efficiency and gain of short antennas have been focused here, in order to determine the importance of those factors affecting the radiation properties of these radiators [12], [14]. These factors are taken into account within the low frequency $(150 - 250\ \mathrm{kHz})$ and medium frequency $(535 - 1705\ \mathrm{kHz})$ broadcast bands.

The most important factors affecting the antenna efficiency are the wire resistance, the insulator equivalent loss resistance and the ground plane equivalent loss resistance.

These factors are responsible of the antenna efficiency, because they dissipate part of the antenna input power and, for this reason, the antenna gain can be much smaller than the antenna directivity.

It was pointed out that the antenna directivity is an antenna natural property, and it depends on the antenna radiation pattern, which is close to the elevation angle cosine function $(\cos\alpha \text{ or } \sin\theta)$ in the case of a short monopole, like a top-loaded antenna (see (116) in Appendix C).

The monopole top-load does not modify the antenna radiation pattern, but it only modifies the antenna current distribution.

In general, insulator equivalent loss resistance is relatively less important than the wire resistance or the ground plane equivalent loss resistance and, for this reason, it can practically be ignored in the efficiency calculation.

Wire resistance can easily be obtained using the high

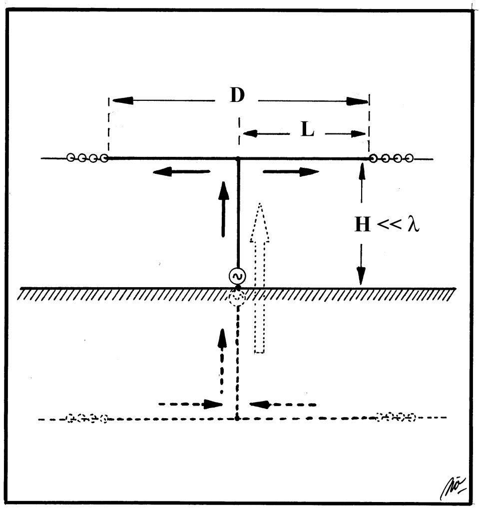

Fig. 2. Sketch of a top-loaded antenna and its wire currents.

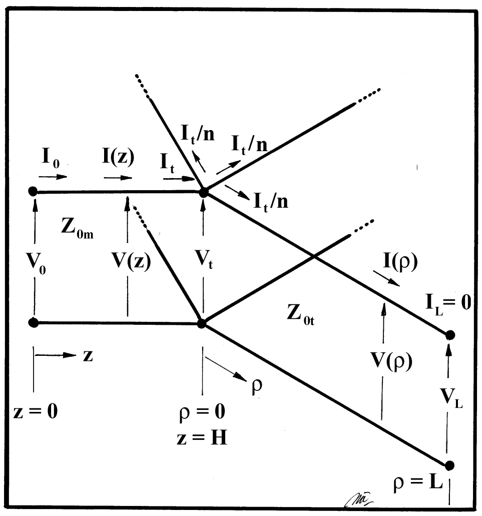

Fig. 3. Sketch of the top-loaded antenna equivalent transmission lines and their currents and voltages.

frequency resistance expression, where the skin effect is taken into account. Under these circumstances, the ground plane equivalent loss resistance is of paramount importance, and its determination constitutes a very difficult task.

In the case of a monopole antenna, the ground plane under it, to be taken into account, is not only the artificial ground plane laid down with metallic radials, but also the natural soil from the radial ends to a distance of half-wavelength from the monopole base or feeding point. Within this half-wavelength radius, the ground plane currents are part of the antenna electric circuit, and they are represented as a current I_0 flowing through the ground plane loss resistance in the antenna equivalent circuit.

The dissipated power is calculated knowing the near tangential magnetic field or the surface current density on this ground plane, whose radius is half-wavelength.

Using the current distribution on the antenna vertical part, the vector magnetic potential is calculated. Near and far magnetic and electric fields are obtained from this potential.

Soil impedance is obtained from the electromagnetic theory and the soil-metallic radials combination impedance using the same technique as Abbott [16]. These parameters, soil resistance and near magnetic field, permit the ground plane dissipated power calculation. Then, the ground plane equivalent loss resistance can be determined from this dissipated power and from the antenna effective input current.

The ground plane equivalent loss resistance is generally not calculated by the standard softwares. Nevertheless, this parameter is very important to calculate the antenna efficiency.

Antenna current distributions on the vertical part and on the top-load are determined quite accurately using the equivalent transmission line theory, and the tip voltages can also be determined from it.

Interesting results were obtained from the determination of the resonant antenna dimensions for different top-loading conditions. These results permit the choice of the more economical antenna or the antenna whose bandwidth and gain are better approaching the transmission necessities. The determination of these parameters permits a clear vision of the antenna possibilities and limitations, according to the antenna height and operation frequency.

II. Top-Loaded Antennas

It is well known that short monopole antennas have a linear current distribution when their heights are lower than $0.15\,\lambda$ and a zero current at the top. Under these conditions, the radiation resistance depends on the following expression, determined from the power density space integration or the antenna total radiated power, divided by the square of the effective input current [4]. Then,

$$\mathrm{R_{rad}} = 40\,\pi^2 \left(\frac{\mathrm{H}}{\lambda}\right)^2 \tag{1}$$

Also

$$\mathrm{R_{rad}} = 10\,(\beta \mathrm{H})^2 \tag{2}$$

Where

$\mathrm{R_{rad}}$ is the antenna radiation resistance $[\Omega]$.

H is the antenna height $[\mathrm{m}]$.

λ is the wavelength $[\mathrm{m}]$.

$\beta = 2\,\pi/\lambda$ is the space phase constant or wave number $[\mathrm{rad/m}]$.

These expressions can be obtained from any antenna book [4], [5], [6], [7].

This radiation resistance is quite small when the ratio H/λ is less than 0.1. This is due to the small area of the current distribution along the antenna, calculated from the base to the top. In this case, the current distribution area is the area of a triangle $\beta\mathrm{H}$ in height and the antenna input current I_0 as the triangle base. The top current $\mathrm{I_t}$ is, of course, a null.

In order to increase the current distribution area, a top-load is used. In this specific case, the top current $\mathrm{I_t}$ is not a null and it depends on the top-loading conditions.

The top-load is built up using one $(\mathrm{n}=1)$ or several $(\mathrm{n}>1)$ branches installed parallel to the surface of the earth and connected to the top of the antenna vertical wire.

If the top-load consists of only one branch $(\mathrm{n}=1)$, an **Inverted-L** antenna is obtained.

If two branches are used $(\mathrm{n}=2)$, a **T** antenna is obtained and if the branch quantity is four $(\mathrm{n}=4)$, the antenna is called an **X** antenna. Otherwise, a **Star** antenna is obtained. This kind of antennas can be seen in Fig. 1.

III. Antenna Input Impedance

In Fig. 2 a sketch of an n-branch top-loaded antenna and its image under the ground plane can be seen. Since currents flowing on the vertical part of the antenna and its image are in phase, it follows that they will produce a net field intensity close and far away in the surrounding space. Currents flowing on the top-load and its image are out of phase and, because of the short distance in wavelengths between them $(2\,\mathrm{H}\ll\lambda)$, the net field intensity in the surrounding space is practically zero and they do not make any contribution to the antenna radiated power.

The current distribution on the antenna vertical wire starts as the base current $\mathrm{I_0}$, at the antenna base, and ends as the top current $\mathrm{I_t}$, at the antenna top. For an n-branch top-loaded antenna, the current distribution on any branch starts as $\mathrm{I_t/n}$, at the antenna top, and, finally, goes down to zero at the tip of the top-load wires, $\mathrm{I_L}=0$.

In order to determine the antenna input impedance the transmission line technique can be used. The equivalent transmission lines, corresponding to the antenna top-load, depend on the antenna type. In Fig. 3 a sketch of the antenna equivalent transmission lines can be seen, where the current and voltage distributions are indicated.

In the Inverted-L antenna case $(\mathrm{n}=1)$, only one transmission line is attached to the top of the antenna vertical wire. In the other cases, there are several transmission lines or branches $(\mathrm{n}>1)$, depending on the antenna type, and they will be connected in parallel to the antenna top.

Each top-load branch transmission line can be made up of one $(\mathrm{n_c}=1)$ or several wires $(\mathrm{n_c}>1)$, taking into account the potential gradient when high power is employed.

The characteristic impedance of the top-load, considered as a transmission line, can be calculated using the logarithmic potential theory. If one wire is used $(\mathrm{n_c}=1)$, the corresponding characteristic impedance $\mathrm{Z_{0t}}$ is given by [3]

$$\mathrm{Z_{0t}}=60\,\ln\left(\frac{2\,\mathrm{H}}{\mathrm{a}}\right)\tag{3}$$

Where
H is the antenna height [m].
a is the wire radius [m].

The antenna vertical wire can also be considered as another transmission line, with an average characteristic impedance $\mathrm{Z_{0m}}$ given by [3]

$$\mathrm{Z_{0m}}=60\,\ln\left(\frac{\mathrm{H}}{\mathrm{a}}\right)\tag{4}$$

For the antenna vertical wire, there are several expressions that can be used to calculate the average characteristic impedance, and all of them give results very close to those obtained here [2], [3].

The input impedance at the top of the antenna, looking along the top-load, is equivalent to the input impedance of n open end low loss transmission lines in parallel. Therefore,

$$\mathrm{Z_t}=\mathrm{j\,X_t}=-\mathrm{j}\,\frac{\mathrm{Z_{0t}}}{\mathrm{n}\,\tan\beta\mathrm{L}}\tag{5}$$

Where
$\mathrm{Z_t}$ is the antenna top impedance $[\Omega]$.
$\mathrm{X_t}$ is the antenna top reactance $[\Omega]$.
$\mathrm{Z_{0t}}$ is the top-load characteristic impedance $[\Omega]$.
L is the top-load length $[\mathrm{m}]$.
n is the number of top-load branches.
$\beta=2\,\pi/\lambda$ is the space phase constant or wave number $[\mathrm{rad/m}]$.

Knowing the antenna top reactance $\mathrm{X_t}$, the antenna top-load capacitance can easily be found as follows

$$\mathrm{C_t}=\frac{1}{2\,\pi\,\mathrm{f}\;|\,\mathrm{X_t}\,|}\quad[\mathrm{F}]\tag{6}$$

Where f is the operation frequency [Hz].

The antenna input impedance $\mathrm{Z_a}$ is equal to the top impedance $\mathrm{Z_t}$ translated to the antenna input terminals.

If the transmission lines are considered to be of low losses, the input impedance will be a pure reactance, nevertheless, the real part of this impedance depends on the antenna radiation resistance and other losses.

Then, the antenna input impedance will be

$$\mathrm{Z_a}=\mathrm{R_a}+\mathrm{j\,X_a}\tag{7}$$

Where
$\mathrm{Z_a}$ is the antenna input impedance $[\Omega]$.
$\mathrm{R_a}$ is the real part of the antenna input impedance $[\Omega]$.
$\mathrm{X_a}$ is the antenna input reactance $[\Omega]$.

The real part of the antenna input impedance, $\mathrm{R_a}$, depends on the antenna radiation resistance $\mathrm{R_{rad}}$ and the equivalent loss resistance $\mathrm{R_{loss}}$. The loss resistance $\mathrm{R_{loss}}$ depends on the conductor resistance $\mathrm{R_c}$, the insulator equivalent loss resistance $\mathrm{R_i}$ and the ground plane equivalent loss resistance $\mathrm{R_{gp}}$.

Therefore,

$$\mathrm{R_a}=\mathrm{R_{rad}}+\mathrm{R_{loss}}\tag{8}$$

Where

$$\mathrm{R_{loss}}=\mathrm{R_c}+\mathrm{R_i}+\mathrm{R_{gp}}\tag{9}$$

In actual cases, using well designed insulators, $\mathrm{R_i}$ is much smaller than the other loss resistances, $\mathrm{R_c}$ and $\mathrm{R_{gp}}$. For this reason, a very small error is introduced in all calculations if $\mathrm{R_i}$ is neglected $(\mathrm{R_i}\cong 0)$.

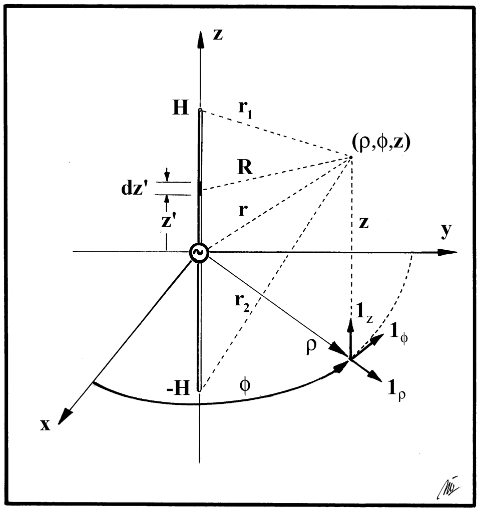

Fig. 4. Antenna geometry used to calculate the electromagnetic field in cylindrical coordinates.

Radiation resistance $\mathrm{R_{rad}}$, conductor resistance $\mathrm{R_c}$ and ground plane equivalent loss resistance $\mathrm{R_{gp}}$ will be determined in Sections VII, VIII and IX, respectively.

Antenna reactance $\mathrm{X_a}$, according to the transmission line theory [8], [9], is given by

$$\mathrm{X_a = Z_{0m}\,\frac{Z_{0m}\ \tan\beta H + X_t}{Z_{0m} - X_t\ \tan\beta H}} \tag{10}$$

Where
$\mathrm{X_a}$ is the antenna input reactance $[\Omega]$.
$\mathrm{Z_{0m}}$ is the antenna average characteristic impedance $[\Omega]$.
$\mathrm{X_t}$ is the antenna top reactance $[\Omega]$.
H is the antenna height $[\mathrm{m}]$.

The top-loaded antenna is resonant if $\mathrm{X_a} = 0$. Under this condition, the top reactance $\mathrm{X_t}$ becomes

$$\mathrm{X_t = -Z_{0m}\ \tan\beta H} \tag{11}$$

Taking into account (5), the length of each top-load branch, for a resonant antenna, becomes

$$\mathrm{L_{res}} = \frac{\lambda}{2\,\pi}\ \arctan\left(\frac{\mathrm{Z_{0t}}}{\mathrm{n\,Z_{0m}\ \tan\beta H}}\right) \tag{12}$$

In most cases, it is very important to built up a self-resonant antenna, because the input voltage is very small compared to a series inductance resonant antenna, where the input voltage is Q times the applied voltage. This condition can be fulfilled choosing a proper top-load length $(\mathrm{L = L_{res}})$ for any antenna height.

It is interesting to notice that the top reactance $\mathrm{X_t}$ and capacitance $\mathrm{C_t}$ of any resonant top-loaded antenna, at a given frequency, are of the same value and independent of the top-load type. They depend only on the self-resonant antenna height.

IV. Antenna Current Distribution

The current distribution on the antenna vertical wire, considered as a piece of transmission line (see Appendix A), will be

$$\mathrm{I(z) = I_0}\left(\cos\beta\mathrm{z} + \frac{\mathrm{X_a}}{\mathrm{Z_{0m}}}\ \sin\beta\mathrm{z}\right) \quad 0 \le \mathrm{z} \le \mathrm{H} \tag{13}$$

The top current, $\mathrm{I_t = I(z = H)}$, is

$$\mathrm{I_t = I_0}\left(\cos\beta\mathrm{H} + \frac{\mathrm{X_a}}{\mathrm{Z_{0m}}}\ \sin\beta\mathrm{H}\right) \tag{14}$$

Then, the top to base current ratio, $\mathrm{I_t/I_0}$, is given by

$$\frac{\mathrm{I_t}}{\mathrm{I_0}} = \cos\beta\mathrm{H} + \frac{\mathrm{X_a}}{\mathrm{Z_{0m}}}\ \sin\beta\mathrm{H} \tag{15}$$

The current distribution on the top-load will be

$$\mathrm{I}(\rho) = \frac{\mathrm{I_t}}{\mathrm{n}}\ \left(\cos\beta\rho - \frac{\sin\beta\rho}{\tan\beta\mathrm{L}}\right) \quad 0 \le \rho \le \mathrm{L} \tag{16}$$

The tip current of the top-load is $\mathrm{I_L} = \mathrm{I}(\rho = \mathrm{L}) = 0$.

In general, a resonant antenna is convenient to be chosen $(\mathrm{X_a} = 0)$ modifying the top-load length to $\mathrm{L = L_{res}}$.

In this case, the current expressions are simplified. Therefore,

$$\mathrm{I(z) = I_0\ \cos\beta z} \quad 0 \le \mathrm{z} \le \mathrm{H} \tag{17}$$

$$\mathrm{I_t = I_0\ \cos\beta H} \tag{18}$$

$$\frac{\mathrm{I_t}}{\mathrm{I_0}} = \cos\beta\mathrm{H} \tag{19}$$

$$\mathrm{I}(\rho) = \frac{\mathrm{I_t}}{\mathrm{n}}\ \left(\cos\beta\rho - \frac{\sin\beta\rho}{\tan\beta\mathrm{L_{res}}}\right) \quad 0 \le \rho \le \mathrm{L_{res}} \tag{20}$$

V. Antenna Voltage Distribution

The voltage distribution is important in order to know the top voltage and the voltage at the tips of the top-load wires. These voltages permit to choose the convenient insulators to support the top wires.

The voltage distribution on the antenna vertical wire is given by (see Appendix A)

$$\mathrm{V(z) = j\,I_0\,(X_a\ \cos\beta z - Z_{0m}\ \sin\beta z)} \quad 0 \le \mathrm{z} \le \mathrm{H} \tag{21}$$

The voltage at the antenna top, $\mathrm{V_t = V(z = H)}$, is

$$\mathrm{V_t = j\,I_0\,(X_a\ \cos\beta H - Z_{0m}\ \sin\beta H)} \tag{22}$$

The voltage distribution on the top-load wires becomes

$$\mathrm{V}(\rho) = \mathrm{j\,I_t}\left(\mathrm{X_t}\ \cos\beta\rho - \frac{\mathrm{Z_{0t}}}{\mathrm{n}}\ \sin\beta\rho\right) \quad 0 \le \rho \le \mathrm{L} \tag{23}$$

Where $\mathrm{I_t}$ is given by (14) and $\mathrm{X_t}$ by (5).

The top-load wire tip voltage, $\mathrm{V_L} = \mathrm{V}(\rho = \mathrm{L})$, becomes

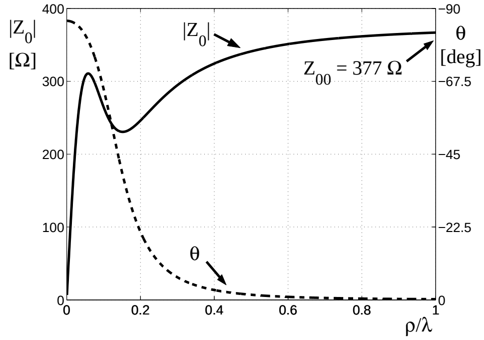

Fig. 5. Inverted-L antenna space impedance at 200 kHz.

$$\mathrm{V_L = j\, I_t \left(X_t\, \cos\beta L - \frac{Z_{0t}}{n}\, \sin\beta L \right)} \tag{24}$$

When the antenna is resonant ($\mathrm{X_a} = 0$), the voltage expressions are simplified. Therefore,

$$\mathrm{V(z) = -j\, I_0\, Z_{0m}\, \sin\beta z} \quad 0 \leq \mathrm{z} \leq \mathrm{H} \tag{25}$$

$$\mathrm{V_t = -j\, I_0\, Z_{0m}\, \sin\beta H} \tag{26}$$

$$\mathrm{V}(\rho) = -\mathrm{j\, I_0\, \cos\beta H} \left(\mathrm{Z_{0m}\, \tan\beta H\, \cos\beta\rho + \frac{Z_{0t}}{n}\, \sin\beta\rho} \right) \tag{27}$$

$$0 \leq \rho \leq \mathrm{L_{res}}$$

$$\mathrm{V_L = -j\, I_0\, \frac{Z_{0t}\, \cos\beta H}{n\, \sin\beta L_{res}}} \tag{28}$$

It is interesting to observe that the tip voltage $\mathrm{V_L}$ decreases as the number of branches n of the top-load increases. For this reason, the Inverted-L antenna has the maximum tip voltage $\mathrm{V_L}$, while the top voltage $\mathrm{V_t}$ is the same for any resonant top-loaded antenna of height H.

VI. Electromagnetic Fields

A. Near Field

The general procedure is used to calculate the electromagnetic fields around a resonant top-loaded antenna. For instance, the magnetic vector potential can be calculated taking into account the current distribution on the antenna vertical wire, $\mathrm{I(z) = I_0\, \cos\beta z}$, because this is the only current that produces the net electromagnetic field.

The antenna geometry used to calculate the electromagnetic field can be seen in Fig. 4, where, as a first approximation, the ground plane conductivity σ is considered to be infinite.

Under this condition, the magnetic vector potential in free space, according to the current distribution, has only one component in the z-direction, that is

$$\mathrm{A_z} = \frac{\mu_0}{4\,\pi} \int_{-\mathrm{H}}^{\mathrm{H}} \mathrm{I(z')}\, \frac{\mathrm{e}^{-\mathrm{j}\beta \mathrm{R}}}{\mathrm{R}}\, \mathrm{dz'} \tag{29}$$

Where $\mathrm{R} = \sqrt{(\mathrm{z - z'})^2 + \rho^2}$ is the distance from the antenna current element $\mathrm{I(z')\,dz'}$ to the observation point (ρ, ϕ, z).

In Appendix B the magnetic vector potential has been obtained, so the magnetic field $\mathbf{H}$ can be calculated using the following classical expression:

$$\mathbf{H} = \frac{1}{\mu_0}\, \mathrm{rot}\, \mathbf{A} \tag{30}$$

In cylindrical coordinates,

$$\mathrm{H}_\phi = -\frac{1}{\mu_0}\, \frac{\partial \mathrm{A_z}}{\partial \rho} \tag{31}$$

Thus, for a resonant top-loaded antenna, the magnetic field intensity on the ground plane, at $\mathrm{z} = 0$, becomes

$$\mathrm{H}_\phi = \frac{\mathrm{I_0}}{2\,\pi}\, \frac{\mathrm{e}^{-\mathrm{j}\beta \mathrm{r_1}}}{\rho} \left(\frac{\mathrm{H}}{\mathrm{r_1}}\, \cos\beta\mathrm{H} + \mathrm{j}\, \sin\beta\mathrm{H} \right) \tag{32}$$

Where $\mathrm{r_1} = \sqrt{\mathrm{H}^2 + \rho^2}$ is the distance from the antenna top to a point on the ground surface, and ρ is the distance from the antenna base to the same point.

This is the magnetic field corresponding to a perfectly conducting ground plane. The actual magnetic field is practically of the same value close to the antenna base, as measurements indicate [17], because it is not appreciably affected by the finite soil conductivity [10], [11].

The electric field intensity $\mathbf{E}$ is obtained by means of the Maxwell equation

$$\mathrm{rot}\, \mathbf{H} = \mathrm{j}\, \omega\, \epsilon_0\, \mathbf{E} \tag{33}$$

Thus, for a resonant top-loaded antenna, the electric field intensity on the earth surface, at $\mathrm{z} = 0$, has only one component $\mathrm{E_z}$ for a perfectly conducting ground plane. Therefore,

$$\mathrm{E_z} = \frac{\mathrm{j\, I_0\, e^{-j\beta r_1}}}{2\,\pi\,\epsilon_0\,\omega} \left(\frac{\mathrm{H}\, \cos\beta\mathrm{H}}{\mathrm{r_1^3}} + \frac{\mathrm{j}\,\beta\mathrm{H}\, \cos\beta\mathrm{H}}{\mathrm{r_1^2}} - \frac{\beta\, \sin\beta\mathrm{H}}{\mathrm{r_1}} \right) \tag{34}$$

The near magnetic and electric fields on the earth surface, H_ϕ and $\mathrm{E_z}$, are clearly more complex functions of the radial distance ρ than in the case of the Hertz monopole, where H is infinitesimal.

When the ground plane is not perfectly conducting, the electric field develops a small radial component E_ρ. This electric field component E_ρ is related to the magnetic field H_ϕ as follows

$$\mathrm{E}_\rho = \begin{cases} -\mathrm{Z_g\, H_\phi} & \text{for } \ 0 < \rho < \mathrm{R_0} \\ -\mathrm{Z_s\, H_\phi} & \text{for } \ \rho > \mathrm{R_0} \end{cases} \tag{35}$$

Where

$\mathrm{Z_g}$ is the parallel impedance of the soil and ground screen $[\Omega]$.

$\mathrm{Z_s}$ is the soil impedance $[\Omega]$.

$\mathrm{R_0}$ is the metallic ground screen radius [m].

The E_ρ and H_ϕ field components produce a wave that propagates into the soil under the antenna and is dissipated as heat.

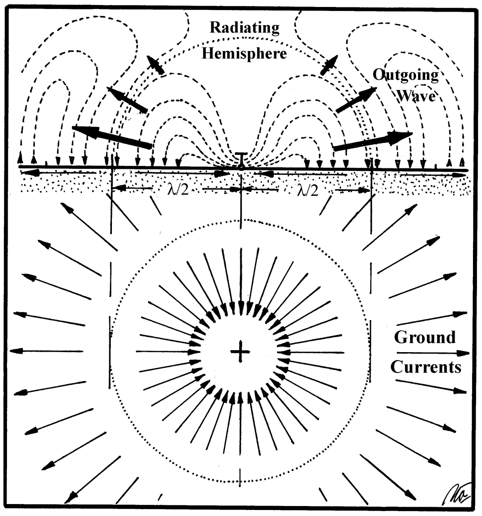

Fig. 6. Sketch of one instant near electric field and ground plane conduction currents.

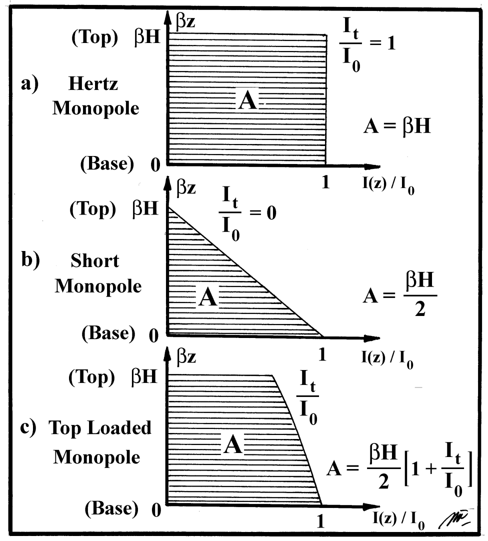

Fig. 7. Top-loaded antenna current distributions and their equivalent areas.

B. Wave Impedance

The ratio between the near electric and magnetic fields, $\mathrm{E_z}$ and H_ϕ, is the wave impedance $\mathrm{Z_0}$ just above the earth surface in the air.

The wave impedance is a complex magnitude, almost purely imaginary very close to the antenna and almost a real magnitude at the distance of half-wavelength from the antenna base. This can be seen as an example in Fig. 5.

This is a good representation of the antenna behavior, and it means that the half-wavelength radius space surrounding the antenna is part of the wave generator (oscillator). The antenna is not only the conductive wires, but a hemispherical free space wave generator half-wavelength in radius.

Through this hemispherical surface, a wave is radiated into the surrounding free space. The earth area under this hemisferical space is a circle, which is very important, because all the conductive currents flowing on it are part of the antenna circuit and, for this reason, it must have the maximum conductivity in order to achieve the maximum antenna efficiency. This circle is half-wavelength in radius, as can be seen in Fig. 6.

For this reason, in order to calculate the antenna efficiency, it is very important to take into account the power dissipated by the near field on this circular surface.

The wave impedance $\mathrm{Z_0}$ is a relationship that changes its real and imaginary parts as a function of the distance from the antenna base. Near to half-wavelength, its value is almost purely real and close to the resistive $377\ \Omega$ of the free space intrinsic impedance $\mathrm{Z_{00}}$. This is an indication of a radiated wave, where the power density is practically real or active, flowing through the hemispherical surface.

C. Far Field

For broadcast use, in low and medium frequencies, field intensity on the surface of the earth is of interest and, as a first approximation, a planar earth can be considered.

Under this condition, the electromagnetic far field intensities can be obtained from the near field expressions (32) and (34), considering that $\rho \gg \mathrm{H}$, $\mathrm{r_1} = \sqrt{\mathrm{H}^2 + \rho^2} \cong \rho$ and neglecting the terms in $1/\rho^2$ and $1/\rho^3$ in favor of the terms in $1/\rho$.

Therefore, for a resonant top-loaded antenna, it follows that

$$\mathrm{H}_\phi = \mathrm{j}\,\frac{\mathrm{I_0}}{2\,\pi}\,\frac{\mathrm{e}^{-\mathrm{j}\beta\rho}}{\rho}\,\sin\beta\mathrm{H} \tag{36}$$

$$\mathrm{E_z} = -\mathrm{j}\,\frac{\beta}{\epsilon_0\,\omega}\,\frac{\mathrm{I_0}}{2\,\pi}\,\frac{\mathrm{e}^{-\mathrm{j}\beta\rho}}{\rho}\,\sin\beta\mathrm{H} \tag{37}$$

Where ρ is the distance from the antenna base to the far field observation point on the earth, at $\mathrm{z} = 0$.

In the case of a very short top-loaded antenna, $\beta\mathrm{H} \ll 1$ and $\sin\beta\mathrm{H} \cong \beta\mathrm{H}$, then the far field expressions are exactly the same of the Hertz monopole.

Taking into account that $\beta/\omega\epsilon_0$ is the free space intrinsic impedance $\mathrm{Z_{00}}$, the electric field intensity will be

$$\mathrm{E_z} = -\mathrm{j}\,\mathrm{Z_{00}}\,\frac{\mathrm{I_0}}{2\,\pi}\,\frac{\mathrm{e}^{-\mathrm{j}\beta\rho}}{\rho}\,\sin\beta\mathrm{H} \tag{38}$$

or

$$\mathrm{E_z} = -\mathrm{Z_{00}}\,\mathrm{H}_\phi \tag{39}$$

Therefore, the obtained vector field intensities are

$$\mathbf{H} = \mathbf{1}_\phi\,\mathrm{H}_\phi \tag{40}$$

$$\mathbf{E} = -\mathbf{1}_\mathrm{z}\,\mathrm{Z_{00}}\,\mathrm{H}_\phi \tag{41}$$

These fields are the no attenuated radiated fields, because they depend only on the inverse distance law. The actual field intensities along the earth are affected by the physical constants of the soil and the diffraction due to the spherical earth [13].

The power density, $\mathbf{P} = (1/2)\,\mathbf{E} \times \mathbf{H}^\star$, becomes

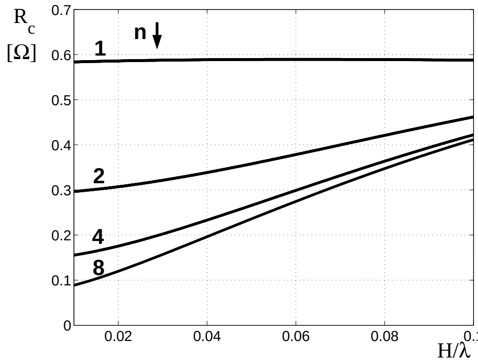

Fig. 8. Wire loss resistance as a function of the resonant antenna height in wavelengths, for different numbers of top-load branches n, at 200 kHz. ($\mathrm{n_c} = 1$, $\mathrm{a} = 6 \cdot 10^{-3}$ m).

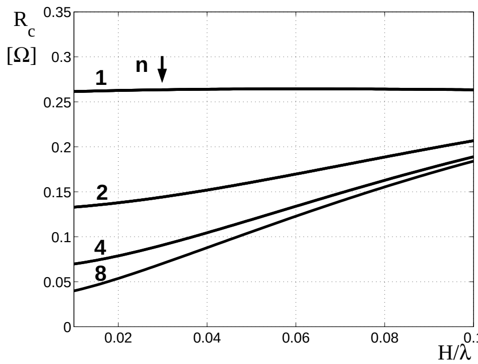

Fig. 9. Wire loss resistance as a function of the resonant antenna height in wavelengths, for different numbers of top-load branches n, at 1 MHz. ($\mathrm{n_c} = 1$, $\mathrm{a} = 6 \cdot 10^{-3}$ m).

$$\mathbf{P} = (-\mathbf{1}_{\mathrm{z}} \times \mathbf{1}_{\phi})\ \frac{1}{2}\ \mathrm{Z}_{00}\ \mid \mathrm{H}_{\phi} \mid^2 \tag{42}$$

or

$$\mathbf{P}\ =\ \mathbf{1}_{\rho}\ \frac{1}{2}\ \mathrm{Z}_{00}\ \mid \mathrm{H}_{\phi} \mid^2 = \mathbf{1}_{\rho}\, \mathrm{P}_{\rho} \tag{43}$$

It can be seen that the power density $\mathbf{P}$ is pointing outward, i.e. the antenna generated wave is an outgoing wave.

In the far field, the ratio between the electric and magnetic field intensities is the free space impedance, $\mathrm{Z}_{00} = 377\ \Omega$. This ratio is clearly obtained at a distance greater than half-wavelength from the antenna base.

In Appendix C the far field expressions in the upper hemisphere and in spherical coordinates are obtained.

VII. Radiation Resistance

The well known Hertz monopole is a top-loaded monopole with a constant current distribution. In this case, the radiation resistance is given by [4]

$$\mathrm{R_{rad}} = 160\,\pi^2 \left(\frac{\mathrm{H}}{\lambda}\right)^2 \tag{44}$$

Also

$$\mathrm{R_{rad}} = 40\,(\beta \mathrm{H})^2 \tag{45}$$

Where
$\mathrm{R_{rad}}$ is the antenna radiation resistance $[\Omega]$.
H is the antenna height $[\mathrm{m}]$.
λ is the wavelength $[\mathrm{m}]$.
$\beta = 2\,\pi/\lambda$ is the space phase constant or wave number $[\mathrm{rad/m}]$.

In Fig. 7.a) the current distribution area of the Hertz monopole can be seen.

If the base current is normalized, $\mathrm{I_0} = 1\,[\mathrm{A}]$, and the antenna height is H in meters or $\beta\mathrm{H}$ in radians, the current distribution area A, normalized to 1 Ampere, is given by

$$\mathrm{A} = \mathrm{I_0}\,\beta \mathrm{H} = \beta \mathrm{H} \tag{46}$$

The radiation resistance of any current distribution is proportional to the square of the area [3], that is

$$\mathrm{R_{rad}} = \mathrm{K}\,\mathrm{A}^2 \tag{47}$$

In the Hertz monopole case, the constant K is equal to 40, because the squared area is $\mathrm{A}^2 = (\beta\mathrm{H})^2$ in (45). Then, for any other antenna, the radiation resistance will be

$$\mathrm{R_{rad}} = 40\,\mathrm{A}^2 \tag{48}$$

In the case of a short monopole with no top-load (Fig. 7.b), the normalized current distribution area is $\mathrm{A} = \beta\mathrm{H}/2$, so the radiation resistance will be

$$\mathrm{R_{rad}} = 40\,\left(\frac{\beta\mathrm{H}}{2}\right)^2 = 10\,(\beta\mathrm{H})^2 \tag{49}$$

For any other top-loaded antenna (Fig. 7.c), the normalized current distribution area is given by

$$\mathrm{A} = \frac{\beta\mathrm{H}}{2}\,\left(1 + \frac{\mathrm{I_t}}{\mathrm{I_0}}\right) \tag{50}$$

Therefore, the radiation resistance for any top-loaded antenna will be

$$\mathrm{R_{rad}} = 40\,\left[\frac{\beta\mathrm{H}}{2}\left(1 + \frac{\mathrm{I_t}}{\mathrm{I_0}}\right)\right]^2 \tag{51}$$

or

$$\mathrm{R_{rad}} = 10\,(\beta\mathrm{H})^2\,\left(1 + \frac{\mathrm{I_t}}{\mathrm{I_0}}\right)^2 \tag{52}$$

Also

$$\mathrm{R_{rad}} = 40\,\pi^2 \left(\frac{\mathrm{H}}{\lambda}\right)^2 \left(1 + \frac{\mathrm{I_t}}{\mathrm{I_0}}\right)^2 \tag{53}$$

This is the radiation resistance expression to be used for any top-loaded antenna, where $\mathrm{I_t}/\mathrm{I_0}$ is given by (15).

If the antenna is resonant, $\mathrm{X_a} = 0$, $\mathrm{L} = \mathrm{L_{res}}$ (12) and $\mathrm{I_t}/\mathrm{I_0} = \cos\beta\mathrm{H}$ (19). Then, the radiation resistance of any resonant top-loaded antenna, including the Hertz monopole ($\beta\mathrm{H} \ll 1$ and $\cos\beta\mathrm{H} \cong 1$), will be

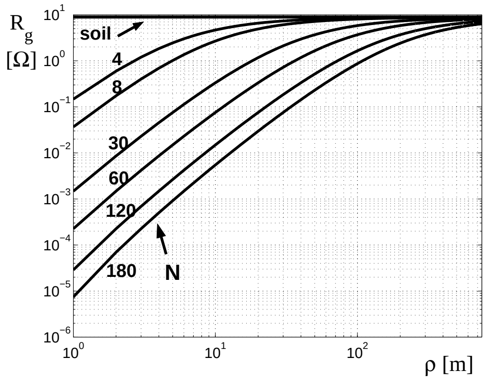

Fig. 10. Artificial ground plane resistance as a function of distance ρ, for different numbers of radials N and over average soil, at 200 kHz. ($\sigma = 10^{-2}$ S/m, $\epsilon_r = 10$).

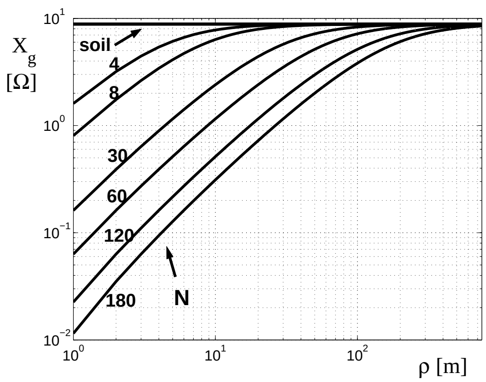

Fig. 11. Artificial ground plane reactance as a function of distance ρ, for different numbers of radials N and over average soil, at 200 kHz. ($\sigma = 10^{-2}$ S/m, $\epsilon_r = 10$).

$$\mathrm{R_{rad}} = 40\pi^2 \left(\frac{\mathrm{H}}{\lambda}\right)^2 (1 + \cos\beta \mathrm{H})^2 \tag{54}$$

This simple expression can successfully be used instead of the exact expression due to a very small error introduced in all calculations.

In Appendix E the exact expression for the radiation resistance has been determined and in Table XV a comparison between both expressions can be seen.

VIII. Wire Loss Resistance

Antenna vertical wire and top-load wires do not have infinite conductivity. For this reason, the conductor or wire loss resistance $\mathrm{R_c}$ must be calculated taking into account the wire conductivity σ_c and the wire equivalent radius a, according to the current distribution on the antenna vertical wire and on the top-load wires.

The wire loss resistance dissipates part of the antenna input power, and can be placed in series with the radiation resistance in the antenna equivalent circuit.

Conductor resistance per unit length $\mathrm{R_l}$ can be calculated by means of the following expression, which takes into account the skin effect [4], that is

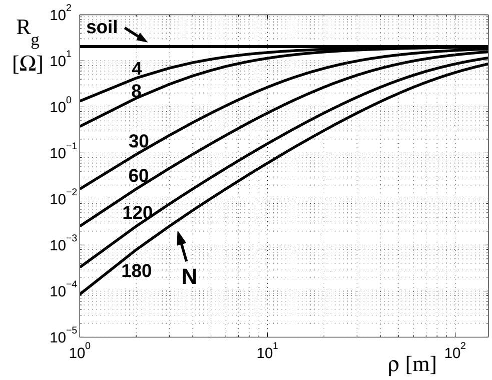

Fig. 12. Artificial ground plane resistance as a function of distance ρ, for different numbers of radials N and over average soil, at 1 MHz. ($\sigma = 10^{-2}$ S/m, $\epsilon_r = 10$).

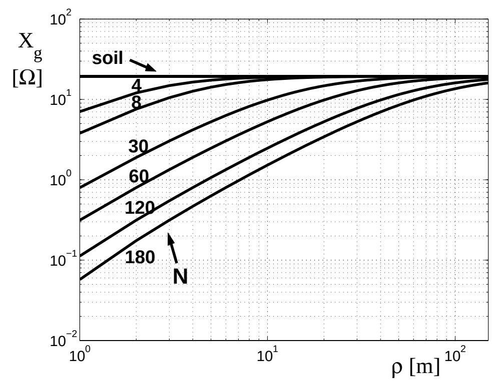

Fig. 13. Artificial ground plane reactance as a function of distance ρ, for different numbers of radials N and over average soil, at 1 MHz. ($\sigma = 10^{-2}$ S/m, $\epsilon_r = 10$).

$$\mathrm{R_l} = \frac{1}{\mathrm{a}} \sqrt{\frac{\mathrm{f}\,\mu_0}{4\,\pi\,\sigma_c}} \tag{55}$$

For a resonant top-loaded antenna, permeability of free space, $\mu_0 = 4\pi \cdot 10^{-7}$ [H/m], and copper conductivity, $\sigma_c \cong 5.8 \cdot 10^7$ [S/m], the last expression becomes

$$\mathrm{R_l} = \frac{4.16}{\mathrm{a}} \sqrt{\mathrm{f}} \cdot 10^{-8} \quad [\Omega/\mathrm{m}] \tag{56}$$

The power dissipated in the resonant antenna conductors, $\mathrm{W_c}$, taking into account the current distribution, is given by

$$\mathrm{W_c} = \frac{1}{2} \int_0^{\mathrm{H}} \mathrm{I}^2(\mathrm{z})\,\mathrm{R_l}\,\mathrm{dz} + \frac{\mathrm{n}}{2} \int_0^{\mathrm{L_{res}}} \mathrm{I}^2(\rho)\,\mathrm{R_l}\,\mathrm{d}\rho \tag{57}$$

The first term of this expression is the power dissipated in the antenna vertical wire, while the second one is the power dissipated in the antenna n top-load branches.

The wire loss resistance $\mathrm{R_c}$ will be

$$\mathrm{R_c} = \frac{2\,\mathrm{W_c}}{\mathrm{I_0^2}} \tag{58}$$

or

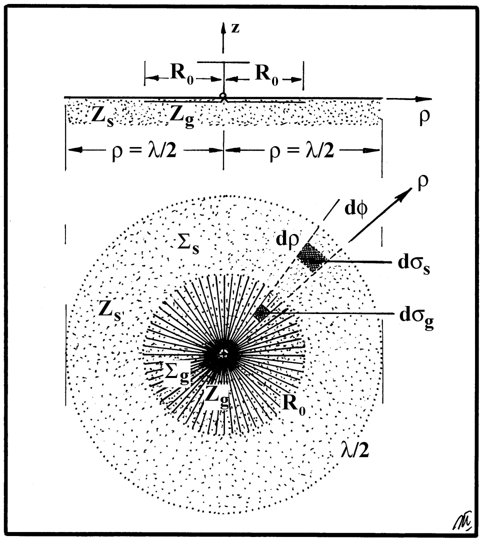

Fig. 14. View of the ground plane used to calculate the soil dissipated power.

$$\mathrm{R_c} = \frac{\mathrm{R_l}}{\mathrm{I_0^2}}\left(\int_0^{\mathrm{H}} \mathrm{I^2(z)\,dz} + \mathrm{n}\int_0^{\mathrm{L_{res}}} \mathrm{I}^2(\rho)\,\mathrm{d}\rho\right) \tag{59}$$

The integrals in this expression can be solved analytically and can be seen in Appendix F.

In Fig. 8 the wire loss resistance $\mathrm{R_c}$ for different resonant top-loaded antennas can be seen as a function of the antenna height at the frequency of $200\ \mathrm{kHz}$ and for a single $(\mathrm{n_c} = 1)$ copper conductor $6\ \mathrm{mm}$ in radius.

Also, in Fig. 9 the wire loss resistance $\mathrm{R_c}$ for different resonant top-loaded antennas can be seen as a function of the antenna height at the frequency of $1\ \mathrm{MHz}$ and for a single $(\mathrm{n_c} = 1)$ copper conductor $6\ \mathrm{mm}$ in radius.

From these figures, it is interesting to observe that, for any height (H/λ), the wire loss resistance for the resonant Inverted-L antenna $(\mathrm{n} = 1)$ is practically constant.

IX. Ground Plane

Previously, it was pointed out that a monopole antenna is equivalent to a hemispherical surface, where the wave generator is placed. This hemispherical surface has a radius of half-wavelength. The corresponding ground plane, where the conduction currents are flowing, is the circular surface of the earth soil. This circular surface is half-wavelength in radius, and it is used to calculate the power dissipated by the conductive currents due to the finite ground conductivity. Within this surface, all the ground currents are part of the antenna electric circuit and, for this reason, this is the area to be taken into account, and not only the surface occupied by the artificial ground plane metallic radials.

Therefore, within this half-wavelength radius circular surface there are two zones,

(a) The artificial ground plane zone [15], [16], where the metallic radials are buried, for the distance ρ varying in the range $0 \le \rho \le \mathrm{R_0}$.

(b) The natural ground plane zone, for the distance ρ varying in the range $\mathrm{R_0} \le \rho \le \lambda/2$.

A. Ground Plane Impedance

From the electromagnetic theory, it is well known that a medium, like the soil, can be a conductor or a dielectric, depending on the ratio $\sigma/\omega\epsilon$, where σ is the soil conductivity, ϵ is the soil permittivity, $\omega = 2\pi\mathrm{f}$ and f is the operation frequency.

For any non-magnetic medium $(\mu = \mu_0)$ like the earth soil, the impedance $\mathrm{Z_s}$ can be calculated from the physical constants, σ and ϵ, as follows [4]

$$\mathrm{Z_s} = \mathrm{R_s} + \mathrm{j\,X_s} = \sqrt{\frac{\mathrm{j}\,\omega\,\mu_0}{\sigma + \mathrm{j}\,\omega\,\epsilon}} \tag{60}$$

This is the impedance of the natural ground plane.

When a star of N conductive radials or ground screen is laid down into the soil, in order to increase the soil conductivity, the impedance of this screen is given by [16]

$$\mathrm{Z_r}(\rho) = \mathrm{j\,X_r} = \mathrm{j\,2\,f}\,\mu_0\,\rho\,\sin\left(\frac{\pi}{\mathrm{N}}\right)\,\ln\left[\frac{\rho}{\pi\,\mathrm{a_0}}\,\sin\left(\frac{\pi}{\mathrm{N}}\right)\right] \tag{61}$$

$$0 \le \rho \le \mathrm{R_0}$$

Where

$\mathrm{Z_r}(\rho)$ is the screen impedance at the distance ρ from the antenna base $[\Omega]$.

f is the operation frequency $[\mathrm{Hz}]$.

ρ is the distance from the star center $[\mathrm{m}]$.

N is the number of radials.

$\mathrm{a_0}$ is the radius of the radial conductors $[\mathrm{m}]$.

$\mathrm{R_0}$ is the radius of the star or ground screen $[\mathrm{m}]$.

Placing both impedances $\mathrm{Z_s}$ and $\mathrm{Z_r}$ in parallel, the artificial ground plane impedance $\mathrm{Z_g}$ will be

$$\mathrm{Z_g} = \mathrm{R_g} + \mathrm{j\,X_g} = \frac{\mathrm{Z_s\,Z_r}}{\mathrm{Z_s} + \mathrm{Z_r}} \tag{62}$$

This expression is valid between the star center, at $\rho = 0$, and the distance $\rho = \mathrm{R_0}$. Beyond this point, the impedance is that of the natural soil $\mathrm{Z_s}$.

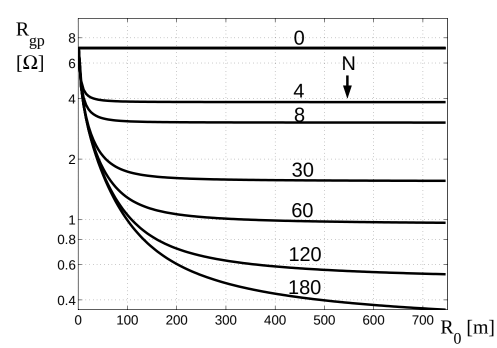

Fig. 15. Ground plane equivalent loss resistance for a resonant Inverted-L antenna over average ground as a function of the artificial ground plane radius $\mathrm{R_0}$ at 200 kHz, and for different radial numbers. $(\mathrm{n} = 1,\ \mathrm{n_c} = 1,\ \mathrm{H} = 105\ \mathrm{m},\ \mathrm{L_{res}} = 276.2\ \mathrm{m},\ \mathrm{a} = 6 \cdot 10^{-3}\ \mathrm{m},\ \sigma = 10^{-2}\ \mathrm{S/m},\ \epsilon_\mathrm{r} = 10)$.

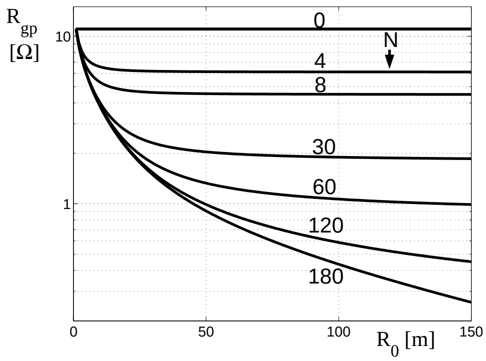

Fig. 16. Ground plane equivalent loss resistance for a resonant Inverted-L antenna over average ground as a function of the artificial ground plane radius $\mathrm{R_0}$ at 1 MHz, and for different radial numbers. ($\mathrm{n = 1,\ n_c = 1,\ H = 21\ m,\ L_{res} = 55.46\ m,\ a = 6 \cdot 10^{-3}\ m,}$ $\sigma = 10^{-2}\ \mathrm{S/m},\ \epsilon_r = 10$).

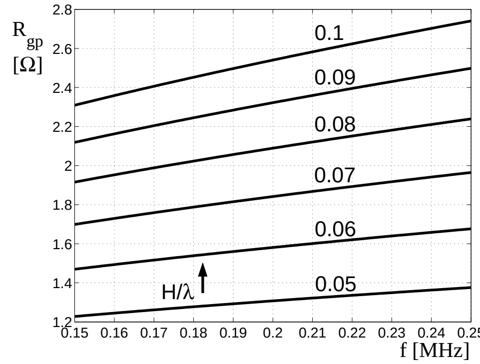

Fig. 17. Ground plane equivalent loss resistance for a resonant Inverted-L antenna as a function of frequency, for different antenna heights and over an average ground, for LF band. ($\mathrm{n = 1,\ n_c = 1,\ a = 6 \cdot 10^{-3}\ m,}$ $\mathrm{R_0 = 0.05\lambda,\ N = 30,}\ \sigma = 10^{-2}\ \mathrm{S/m},\ \epsilon_r = 10$).

In Figs. 10 and 11 an example of the artificial ground plane impedance can be seen for the LF band center frequency of 200 kHz.

In Figs. 12 and 13 an example of the artificial ground plane impedance can be seen for the MF band center frequency of 1 MHz.

It is interesting to note that the artificial ground plane impedance, $\mathrm{Z_g = R_g + j\,X_g}$, is increasing as the distance ρ is increasing, due to the divergence of the star wires.

B. Ground Plane Power Loss

The antenna currents flowing on the ground plane are dissipating power, due to the finite soil conductivity or losses. This dissipated power can be calculated knowing the near magnetic field **H**, which is equal to the surface current density $\mathbf{J}_{\mathrm{su}}$ on the soil.

In Fig. 14 a view of the ground plane used to calculate the ground plane dissipated power can be seen. The ground plane dissipating power surface is made up by two surfaces, the artificial ground plane surface Σ_g with the buried metallic radials, and the natural ground plane surface Σ_s up to a distance ρ of half-wavelength.

Thus, the ground plane dissipated power is given by

$$\mathrm{W_d} = \frac{1}{2}\int_{\Sigma_\mathrm{g}} \mid \mathrm{J_{su}} \mid^2\ \mathrm{R_g}\, \mathrm{d}\sigma_\mathrm{g} + \frac{1}{2}\int_{\Sigma_\mathrm{s}} \mid \mathrm{J_{su}} \mid^2\ \mathrm{R_s}\, \mathrm{d}\sigma_\mathrm{s} \quad (63)$$

Where

$\mathrm{W_d}$ is the power dissipated in the ground plane [W].

$\mathrm{J_{su}}$ is the surface current density $[\mathrm{A/m}]$.

Σ_g is the surface of the artificial ground plane for $0 \le \rho \le \mathrm{R_0}$.

$\mathrm{R_g}$ is the real part of the artificial ground plane impedance $\mathrm{Z_g}$ $[\Omega]$.

Σ_s is the surface of the natural ground plane for $\mathrm{R_0} \le \rho \le \lambda/2$.

$\mathrm{R_s}$ is the real part of the natural ground plane impedance $\mathrm{Z_s}$ $[\Omega]$.

The surface current density $\mathrm{J_{su}}$ is equal to the near magnetic field H_ϕ (32) on the artificial and natural ground planes.

Then, taking into account the cylindrical symmetry, the dissipated power becomes

$$\mathrm{W_d} = \pi\left(\int_0^{\mathrm{R_0}} \mid \mathrm{H}_\phi \mid^2\ \mathrm{R_g}\,\rho\,\mathrm{d}\rho + \int_{\mathrm{R_0}}^{\lambda/2} \mid \mathrm{H}_\phi \mid^2\ \mathrm{R_s}\,\rho\,\mathrm{d}\rho\right) \quad (64)$$

The first integral is the power dissipated in the artificial ground plane surface Σ_g and the second one is the power dissipated in the natural ground plane surface Σ_s.

C. Ground Plane Equivalent Loss Resistance

The ground plane equivalent loss resistance $\mathrm{R_{gp}}$ is necessary to be known, because it is an important factor in the antenna electric circuit, and it permits the calculation of the total equivalent loss resistance $\mathrm{R_{loss}}$ in (9).

The ground plane equivalent loss resistance $\mathrm{R_{gp}}$ is given by the ratio between the power dissipated in the ground plane and the square of the antenna effective input current. Therefore,

$$\mathrm{R_{gp}} = \frac{2\,\mathrm{W_d}}{\mathrm{I_0^2}} \quad (65)$$

Where $\mathrm{I_0}$ is the peak value of the antenna input current.

From (64) and (65), it follows that

$$\mathrm{R_{gp}} = \frac{2\pi}{\mathrm{I_0^2}}\left(\int_0^{\mathrm{R_0}} \mid \mathrm{H}_\phi \mid^2\ \mathrm{R_g}\,\rho\,\mathrm{d}\rho + \int_{\mathrm{R_0}}^{\lambda/2} \mid \mathrm{H}_\phi \mid^2\ \mathrm{R_s}\,\rho\,\mathrm{d}\rho\right) \quad (66)$$

The first integral cannot be evaluated in closed form, and it will be resolved numerically. The second one can be evaluated analytically and is given in Appendix G.

The ground plane equivalent loss resistance $\mathrm{R_{gp}}$ depends on the antenna height H, the number of radials N, the radius $\mathrm{R_0}$ of the artificial ground plane and the physical constants of the soil under the antenna.

In Figs. 15 and 16 the ground plane equivalent loss resistance $\mathrm{R_{gp}}$ has been calculated for a resonant Inverted-L antenna. As an example, this resistance can be seen as a function of the artificial ground plane radius $\mathrm{R_0}$, for different radial numbers N, at the frequencies of 200 kHz and 1 MHz

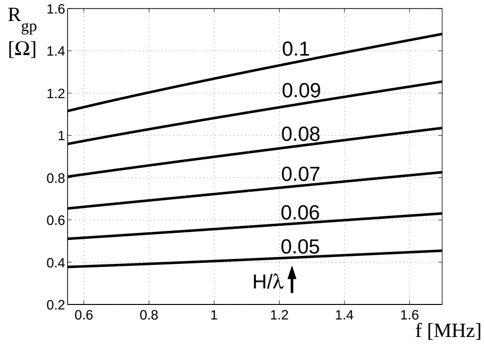

Fig. 18. Ground plane equivalent loss resistance for a resonant Inverted-L antenna as a function of frequency, for different antenna heights and over an average ground, for MF band. $(\mathrm{n}=1,\ \mathrm{n_c}=1,\ \mathrm{a}=6\cdot10^{-3}\ \mathrm{m},\ \mathrm{R_0}=0.25\lambda,\ \mathrm{N}=120,\ \sigma=10^{-2}\ \mathrm{S/m},\ \epsilon_\mathrm{r}=10)$.

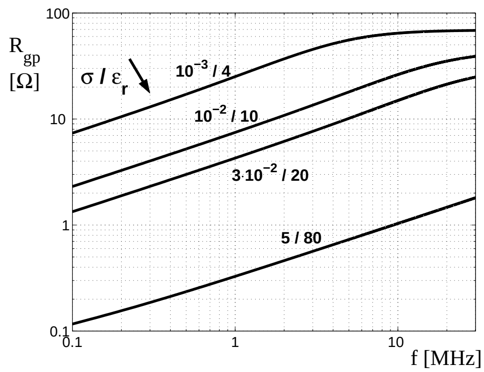

Fig. 19. Ground plane equivalent loss resistance for a resonant Inverted-L antenna as a function of frequency and for different soil physical conditions. $(\mathrm{n}=1,\ \mathrm{n_c}=1,\ \mathrm{H}=0.07\lambda,\ \mathrm{a}=6\cdot10^{-3}\ \mathrm{m},\ \mathrm{R_0}=0.01\lambda,\ \mathrm{N}=180)$.

and over average ground. These frequencies are representative of the low $(150-250\ \mathrm{kHz})$ and medium $(535-1705\ \mathrm{kHz})$ broadcast AM bands. In these figures, $\mathrm{N}=0$ corresponds to the bare soil.

In Figs. 17 and 18 the ground plane equivalent loss resistance $\mathrm{R_{gp}}$ has been calculated for a resonant Inverted-L antenna, as a function of frequency for different antenna heights, over an average ground, and for the low $(150-250\ \mathrm{kHz})$ and medium $(535-1705\ \mathrm{kHz})$ frequency bands.

In Fig. 19 the ground plane equivalent loss resistance $\mathrm{R_{gp}}$ has been calculated for a resonant Inverted-L antenna $(\mathrm{H}=0.07\lambda)$, over a very small artificial ground plane $(\mathrm{R_0}=0.01\lambda)$, as a function of frequency and for different soil physical conditions. This is a good representation of the effect of the soil on the ground plane equivalent loss resistance $\mathrm{R_{gp}}$ to be included in the antenna equivalent series circuit, when practically no artificial ground plane is used.

X. Resonant Antenna Input Resistance

Antenna input resistance $\mathrm{R_a}$ can be calculated as the sum of the three main antenna resistances, or radiation resistance $\mathrm{R_{rad}}$, wire loss resistance $\mathrm{R_c}$ and ground plane equivalent loss resistance $\mathrm{R_{gp}}$ $(\mathrm{R_a}=\mathrm{R_{rad}}+\mathrm{R_c}+\mathrm{R_{gp}})$.

In Figs. 20 and 21 the resonant X antenna input resistance has been calculated for the frequencies of $200\ \mathrm{kHz}$ and $1\ \mathrm{MHz}$,

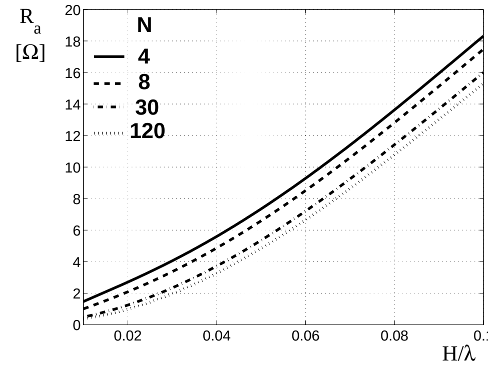

Fig. 20. Resonant X antenna input resistance as a function of the antenna height and for different numbers of radials N at 200 kHz. $(\mathrm{n}=4,\ \mathrm{n_c}=1,\ \mathrm{a}=6\cdot10^{-3}\ \mathrm{m},\ \mathrm{R_0}=0.05\lambda,\ \mathrm{a_0}=1.5\cdot10^{-3}\ \mathrm{m},\ \sigma=10^{-2}\ \mathrm{S/m},\ \epsilon_\mathrm{r}=10)$.

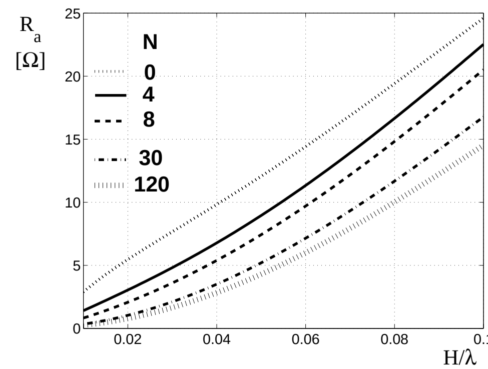

Fig. 21. Resonant X antenna input resistance as a function of the antenna height and for different numbers of radials N at 1 MHz. $(\mathrm{n}=4,\ \mathrm{n_c}=1,\ \mathrm{a}=6\cdot10^{-3}\ \mathrm{m},\ \mathrm{R_0}=0.25\lambda,\ \mathrm{a_0}=1.5\cdot10^{-3}\ \mathrm{m},\ \sigma=10^{-2}\ \mathrm{S/m},\ \epsilon_\mathrm{r}=10)$.

as a function of the antenna height, for different artificial ground plane radial numbers and average soil.

In the medium frequency case, the zero radial number result corresponds to the bare soil. In the low frequency case, the zero and four radial numbers have practically the same result.

In Fig. 22 an example of the antenna equivalent series electric circuit can be seen for a resonant X antenna, with a quarter-wave 120 radials artificial ground plane over average soil, at a frequency of $1\ \mathrm{MHz}$.

The calculated radiation resistance is $\mathrm{R_{rad}}=7.02\ \Omega$, conductor resistance $\mathrm{R_c}=0.15\ \Omega$, ground plane equivalent loss resistance $\mathrm{R_{gp}}=0.72\ \Omega$, efficiency $\eta=0.89$, gain $\mathrm{G}=4.26\ \mathrm{dBi}$, an effective current $\mathrm{I_{0ef}}=11.3\ \mathrm{A}$ and input voltage $\mathrm{V_{0ef}}=88.5\ \mathrm{V}$ for an input power $\mathrm{W_{in}}=1\ \mathrm{kW}$.

XI. Antenna Efficiency and Gain

Antenna efficiency η is determined as the ratio between the antenna radiated power $\mathrm{W_{rad}}$ and the antenna input power $\mathrm{W_{in}}$. Both powers are calculated using the square of the antenna effective input current. Therefore,

$$\eta=\frac{\mathrm{W_{rad}}}{\mathrm{W_{in}}} \tag{67}$$

Then

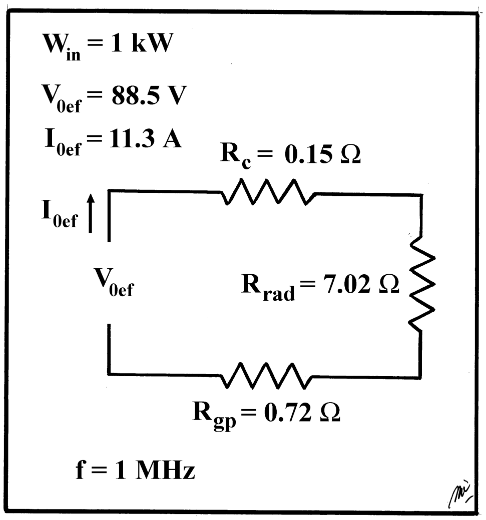

Fig. 22. Resonant X antenna equivalent circuit at 1 MHz. ($n = 4$, $n_c = 1$, $H = 21$ m, $L_{res} = 25$ m, $a = 6 \cdot 10^{-3}$ m, $R_0 = 75$ m, $N = 120$, $a_0 = 1.5 \cdot 10^{-3}$ m, $\sigma = 10^{-2}$ S/m, $\epsilon_r = 10$).

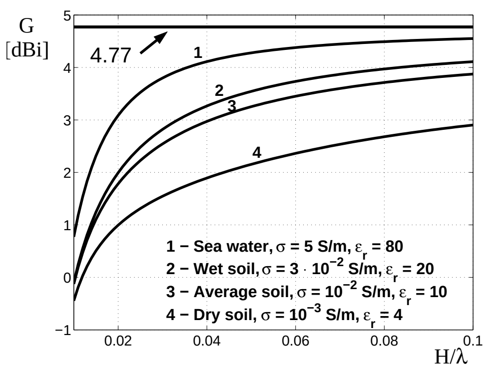

Fig. 23. Resonant X antenna gain as a function of the antenna height and for several soil conditions at 200 kHz. ($n = 4$, $n_c = 1$, $a = 6 \cdot 10^{-3}$ m, $R_0 = 75$ m, $N = 30$, $a_0 = 1.5 \cdot 10^{-3}$ m).

$$\eta = \frac{R_{rad}}{R_a} \tag{68}$$

Where $R_a = R_{rad} + R_c + R_{gp}$ is the antenna input resistance, and the insulator equivalent loss resistance R_i has been neglected.

The monopole antenna directivity D depends on the antenna far field radiation pattern and, in spherical coordinates, it is defined as

$$D = \frac{4\,\pi\,P_{max}}{\int_0^{2\pi} d\phi \int_0^{\pi/2} P(\theta,\phi)\,\sin\theta\,d\theta} \tag{69}$$

Where P is the antenna radiated power density.

For a short top-loaded antenna, the directivity D is practically equal to 3 or $D = 4.77$ dBi (see Appendix D).

The antenna gain G depends on the directivity D and efficiency η, and it is given by

$$G = \eta\,D \tag{70}$$

These antenna efficiency and gain have been calculated for several top-loaded antennas, for different frequencies and soil conditions, in both the low and medium frequency broadcast bands.

As an example, in Fig. 23 the resonant X antenna gain was calculated at the frequency of 200 kHz as a function of the antenna height H/λ and for several soil conditions. In low frequency band, the artificial ground plane radius R_0 is equal to 0.05λ ($R_0 = 75$ m at 200 kHz) and the number of radials is $N = 30$.

In Table I the resonant Inverted-L antenna efficiency and gain, for $H = 0.07\lambda$, have been calculated at the frequency of 200 kHz and for several soil conditions.

In Table II the resonant T antenna efficiency and gain, for $H = 0.07\lambda$, have been calculated at the frequency of 200 kHz and for several soil conditions.

In Table III the resonant X antenna efficiency and gain, for $H = 0.07\lambda$, have been calculated at the frequency of 200 kHz and for several soil conditions.

In Fig. 24 the resonant X antenna gain can be seen as a function of the antenna height, for different radial numbers N and over average ground at 200 kHz.

In Fig. 25 a resonant X antenna gain was calculated at the frequency of 1 MHz as a function of the antenna height H/λ and for several soil conditions. In medium frequency band, the artificial ground plane radius R_0 is equal to 0.25λ

TABLE I
INVERTED-L ANTENNA EFFICIENCY AND GAIN.
$f = 200$ kHz, $H = 105$ m, $R_0 = 75$ m, $N = 30$.

σ	ϵ_r	η	G
S/m	—	—	dBi
10^{-3}	4	0.585	2.44
10^{-2}	10	0.743	3.48
$3 \cdot 10^{-2}$	20	0.789	3.74
5	80	0.897	4.30

TABLE II
T ANTENNA EFFICIENCY AND GAIN.
$f = 200$ kHz, $H = 105$ m, $R_0 = 75$ m, $N = 30$.

σ	ϵ_r	η	G
S/m	—	—	dBi
10^{-3}	4	0.594	2.51
10^{-2}	10	0.758	3.57
$3 \cdot 10^{-2}$	20	0.806	3.84
5	80	0.919	4.41

TABLE III
X ANTENNA EFFICIENCY AND GAIN.
$f = 200$ kHz, $H = 105$ m, $R_0 = 75$ m, $N = 30$.

σ	ϵ_r	η	G
S/m	—	—	dBi
10^{-3}	4	0.598	2.54
10^{-2}	10	0.764	3.60
$3 \cdot 10^{-2}$	20	0.813	3.87
5	80	0.928	4.45

TABLE IV

INVERTED-L ANTENNA EFFICIENCY AND GAIN.

$f = 1.00$ MHz, $H = 21$ m, $R_0 = 75$ m, $N = 120$.

σ	ϵ_r	η	G
S/m	—	—	dBi
10^{-3}	4	0.777	3.67
10^{-2}	10	0.877	4.20
$3 \cdot 10^{-2}$	20	0.896	4.29
5	80	0.940	4.50

TABLE V

T ANTENNA EFFICIENCY AND GAIN.

$f = 1.00$ MHz, $H = 21$ m, $R_0 = 75$ m, $N = 120$.

σ	ϵ_r	η	G
S/m	—	—	dBi
10^{-3}	4	0.784	3.71
10^{-2}	10	0.886	4.25
$3 \cdot 10^{-2}$	20	0.906	4.34
5	80	0.951	4.55

$(R_0 = 75 \text{ m at } 1 \text{ MHz})$ and the number of radials is $N = 120$.

In Table IV the resonant Inverted-L antenna efficiency and gain, for $H = 0.07\lambda$, have been calculated at the frequency of 1 MHz and for several soil conditions.

In Table V the resonant T antenna efficiency and gain, for $H = 0.07\lambda$, have been calculated at the frequency of 1 MHz and for several soil conditions.

In Table VI the resonant X antenna efficiency and gain, for $H = 0.07\lambda$, have been calculated at the frequency of 1 MHz and for several soil conditions.

The small gain increase, in both low and medium frequency bands, of the T and X antennas compared to the Inverted-L, is due to the decreased wire loss resistance of the top-load wires. This wire loss resistance decrease is due to the increase of the top-load branches n, so the top-load current is divided accordingly.

It is important to understand that the wire dissipated power is proportional to the square of the current, so the smaller the current, the much smaller the wire dissipated power and the wire loss resistance.

The relative smaller gain of the top-loaded antenna in the low frequency band, compared to the medium frequency band, is due to the better artificial ground plane used in the latter.

In Fig. 26 the resonant X antenna gain can be seen as a function of the antenna height and for different radial numbers N, over average ground at 1 MHz.

TABLE VI

X ANTENNA EFFICIENCY AND GAIN.

$f = 1.00$ MHz, $H = 21$ m, $R_0 = 75$ m, $N = 120$.

σ	ϵ_r	η	G
S/m	—	—	dBi
10^{-3}	4	0.787	3.73
10^{-2}	10	0.890	4.26
$3 \cdot 10^{-2}$	20	0.909	4.36
5	80	0.955	4.57

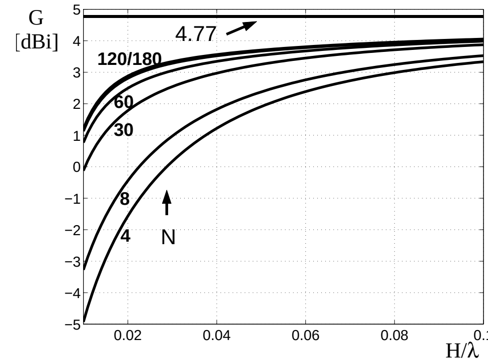

Fig. 24. Resonant X antenna gain as a function of the antenna height, for different numbers of radials N and over average ground at 200 kHz. ($n = 4$, $n_c = 1$, $a = 6 \cdot 10^{-3}$ m, $R_0 = 75$ m, $a_0 = 1.5 \cdot 10^{-3}$ m, $\sigma = 10^{-2}$ S/m, $\epsilon_r = 10$).

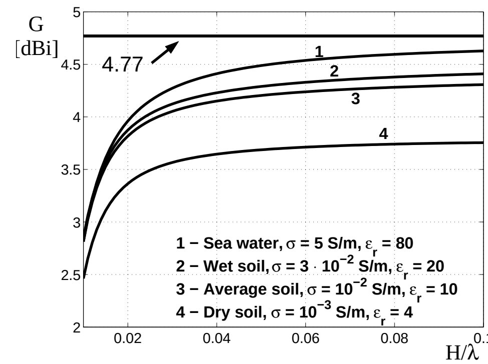

Fig. 25. Resonant X antenna gain as a function of the antenna height and for several soil conditions at 1 MHz. ($n = 4,\ n_c = 1,\ a = 6 \cdot 10^{-3}$ m, $R_0 = 75$ m, $N = 120,\ a_0 = 1.5 \cdot 10^{-3}$ m).

In low frequency band $(150 - 250 \text{ kHz})$, resonant X antenna gain has been calculated as a function of number and length of radials, as it can be seen in Tables VII, VIII and IX for different soil conditions. This gain is quite similar (within 0.5 dB) in the cases of the Inverted-L and T antennas, and these tables can be taken as a good reference.

In the case of dry soil ($\sigma = 10^{-3}$ S/m, $\epsilon_r = 4$), it can be seen a gain increase of around 1 dB changing the artificial ground plane from $R_0 = 0.05\lambda$ and $N = 30$ to $R_0 = 0.15\lambda$ and $N = 120$.

It can be appreciated from these calculations that the increase of the antenna gain is around 0.5 dB increasing the ground plane from $R_0 = 0.05\lambda$ and $N = 30$ to $R_0 = 0.15\lambda$ and $N = 120$ for an average soil ($\sigma = 10^{-2}$ S/m, $\epsilon_r = 10$).

In the case of wet soil ($\sigma = 3 \cdot 10^{-2}$ S/m, $\epsilon_r = 20$), the improvement in gain is less than 0.5 dB increasing the ground plane in the same manner. In these last cases, the gain improvement is quite small and it possibly does not pay the investment in labor and materials, and it must carefully be analyzed.

In medium frequency band $(535 - 1705 \text{ kHz})$, resonant X antenna gain has been calculated as a function of number N and length of radials R_0, as it can be seen in Tables X, XI

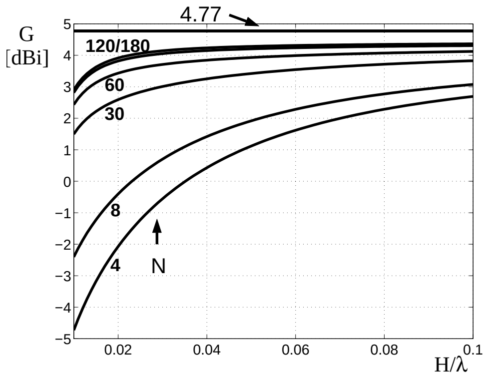

Fig. 26. Resonant X antenna gain as a function of the antenna height, for different numbers of radials N and over average ground at 1 MHz. ($n = 4$, $n_c = 1$, $a = 6 \cdot 10^{-3}$ m, $R_0 = 75$ m, $a_0 = 1.5 \cdot 10^{-3}$ m, $\sigma = 10^{-2}$ S/m, $\epsilon_r = 10$).

TABLE VII
X ANTENNA GAIN G [dBi].
$f = 200$ kHz, $H = 105$ m, $\sigma = 10^{-3}$ S/m, $\epsilon_r = 4$.

R_0/λ	N = 4	N = 8	N = 30	N = 60	N = 120	N = 180
0.01	0.49	0.63	0.71	0.71	0.71	0.71
0.05	1.21	1.77	2.54	2.67	2.71	2.72
0.10	1.29	1.92	3.03	3.34	3.46	3.49
0.15	1.30	1.96	3.18	3.57	3.76	3.81
0.20	1.31	1.97	3.24	3.69	3.92	3.99
0.25	1.31	1.98	3.28	3.75	4.03	4.11
0.50	1.32	1.99	3.33	3.88	4.24	4.38

TABLE VIII
X ANTENNA GAIN G [dBi].
$f = 200$ kHz, $H = 105$ m, $\sigma = 10^{-2}$ S/m, $\epsilon_r = 10$.

R_0/λ	N = 4	N = 8	N = 30	N = 60	N = 120	N = 180
0.01	2.59	2.77	2.95	2.97	2.97	2.97
0.05	2.73	3.04	3.60	3.78	3.87	3.89
0.10	2.74	3.06	3.69	3.95	4.11	4.16
0.15	2.74	3.06	3.72	3.99	4.18	4.25
0.20	2.75	3.07	3.72	4.01	4.22	4.29
0.25	2.75	3.07	3.73	4.02	4.23	4.32
0.50	2.75	3.07	3.73	4.03	4.26	4.36

TABLE IX
X ANTENNA GAIN G [dBi].
$f = 200$ kHz, $H = 105$ m, $\sigma = 3 \cdot 10^{-2}$ S/m, $\epsilon_r = 20$.

R_0/λ	N = 4	N = 8	N = 30	N = 60	N = 120	N = 180
0.01	3.20	3.35	3.54	3.57	3.58	3.58
0.05	3.26	3.46	3.87	4.03	4.12	4.15
0.10	3.26	3.47	3.91	4.10	4.24	4.29
0.15	3.26	3.47	3.91	4.12	4.27	4.33
0.20	3.26	3.48	3.92	4.12	4.28	4.35
0.25	3.26	3.48	3.92	4.13	4.29	4.36
0.50	3.26	3.48	3.92	4.13	4.30	4.38

TABLE X
X ANTENNA GAIN G [dBi].
$f = 1$ MHz, $H = 21$ m, $\sigma = 10^{-3}$ S/m, $\epsilon_r = 4$.

R_0/λ	N = 4	N = 8	N = 30	N = 60	N = 120	N = 180
0.01	-1.92	-1.87	-1.85	-1.85	-1.85	-1.85
0.05	-0.10	0.50	1.01	1.05	1.06	1.07
0.10	0.19	1.08	2.21	2.37	2.42	2.42
0.15	0.26	1.23	2.69	2.97	3.06	3.08
0.20	0.29	1.29	2.93	3.32	3.45	3.48
0.25	0.30	1.32	3.08	3.55	3.73	3.77
0.50	0.32	1.37	3.37	4.07	4.47	4.58

TABLE XI
X ANTENNA GAIN G [dBi].
$f = 1$ MHz, $H = 21$ m, $\sigma = 10^{-2}$ S/m, $\epsilon_r = 10$.

R_0/λ	N = 4	N = 8	N = 30	N = 60	N = 120	N = 180
0.01	1.36	1.50	1.57	1.58	1.58	1.58
0.05	1.93	2.41	3.09	3.21	3.25	3.26
0.10	1.98	2.52	3.47	3.73	3.84	3.86
0.15	1.99	2.55	3.57	3.91	4.07	4.11
0.20	1.99	2.55	3.62	3.99	4.19	4.25
0.25	2.00	2.56	3.64	4.04	4.26	4.33
0.50	2.00	2.57	3.68	4.12	4.42	4.53

and XII for different soil conditions.

It can be appreciated from these calculations that the increase of the antenna gain is less than 1 dB increasing the ground plane from $R_0 = 0.25\lambda$ and $N = 30$ to $R_0 = 0.25\lambda$ and $N = 120$ for dry soil ($\sigma = 10^{-3}$ S/m, $\epsilon_r = 4$).

From $N = 60$ to $N = 120$ ($R_0 = 0.25\lambda$), the increase in gain is very small, around 0.2 dB for every soil and, for this reason, the 120 radial case can be considered an optimum artificial ground plane, especially for frequencies higher than 1 MHz.

Also, from these tables, an increase in the artificial ground plane radius R_0 more than 0.25λ is really not necessary, as it can be appreciated, due to a very small increase in gain, less than 0.5 dB. The same can be said for an increase in radial number from $N = 120$ to $N = 180$.

In the case of very dry soil, it must be used the largest artificial ground plane as possible or maximum radius R_0 and number of radials N, taking into account that the antenna

TABLE XII
X ANTENNA GAIN G [dBi].
$f = 1$ MHz, $H = 21$ m, $\sigma = 3 \cdot 10^{-2}$ S/m, $\epsilon_r = 20$.

R_0/λ	N = 4	N = 8	N = 30	N = 60	N = 120	N = 180
0.01	2.36	2.52	2.64	2.65	2.65	2.65
0.05	2.62	2.99	3.59	3.74	3.79	3.80
0.10	2.64	3.03	3.76	4.02	4.15	4.18
0.15	2.65	3.04	3.81	4.10	4.27	4.32
0.20	2.65	3.04	3.82	4.13	4.32	4.39
0.25	2.65	3.05	3.83	4.15	4.36	4.43
0.50	2.65	3.05	3.85	4.18	4.42	4.52

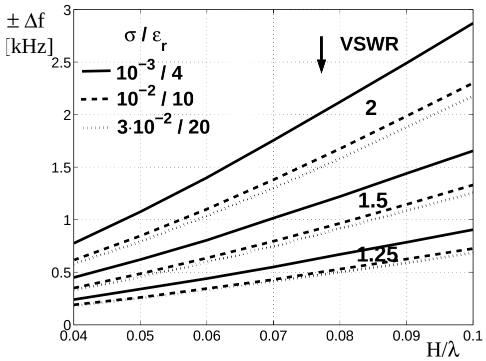

Fig. 27. Resonant Inverted-L antenna bandwidth as a function of the antenna height and for different soil conditions at 200 kHz. ($\mathrm{n} = 1$, $\mathrm{n_c} = 1$, $\mathrm{a} = 6 \cdot 10^{-3}$ m, $\mathrm{R_0} = 0.05\lambda$, $\mathrm{N} = 30$, $\mathrm{a_0} = 1.5 \cdot 10^{-3}$ m).

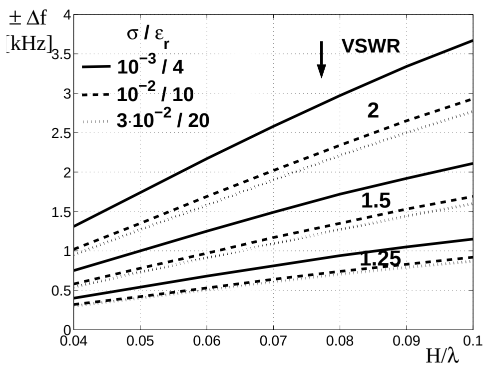

Fig. 28. Resonant T antenna bandwidth as a function of the antenna height and for different soil conditions at 200 kHz. ($\mathrm{n} = 2$, $\mathrm{n_c} = 1$, $\mathrm{a} = 6 \cdot 10^{-3}$ m, $\mathrm{R_0} = 0.05\lambda$, $\mathrm{N} = 30$, $\mathrm{a_0} = 1.5 \cdot 10^{-3}$ m).

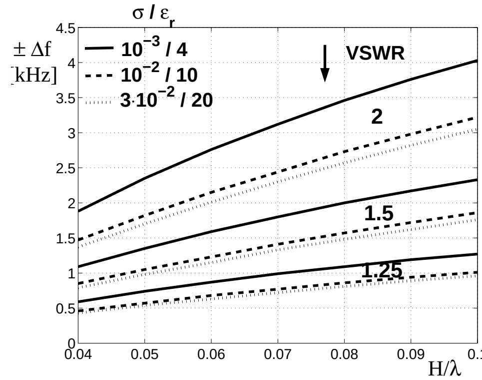

Fig. 29. Resonant X antenna bandwidth as a function of the antenna height and for different soil conditions at 200 kHz. ($\mathrm{n} = 4$, $\mathrm{n_c} = 1$, $\mathrm{a} = 6 \cdot 10^{-3}$ m, $\mathrm{R_0} = 0.05\lambda$, $\mathrm{N} = 30$, $\mathrm{a_0} = 1.5 \cdot 10^{-3}$ m).

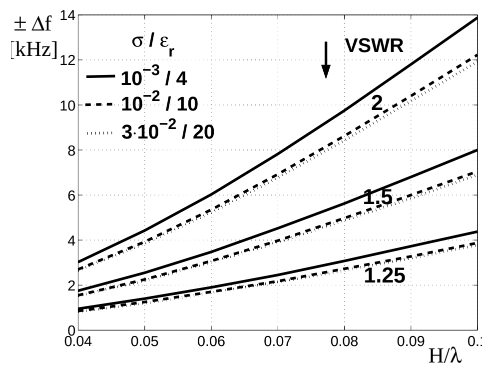

Fig. 30. Resonant Inverted-L antenna bandwidth as a function of the antenna height and for different soil conditions at 1 MHz. ($\mathrm{n} = 1$, $\mathrm{n_c} = 1$, $\mathrm{a} = 6 \cdot 10^{-3}$ m, $\mathrm{R_0} = 0.25\lambda$, $\mathrm{N} = 120$, $\mathrm{a_0} = 1.5 \cdot 10^{-3}$ m).

ground plane currents are reaching a maximum distance of half-wavelength. This is not so important for high conductivity soils, because the increase in gain is very small from the optimum artificial ground plane.

XII. Antenna Bandwidth

The antenna bandwidth is defined according to a maximum value of the reflection coefficient $\Gamma_{\max}$ or maximum standing wave ratio (VSWR) presented by the antenna input impedance within the bandpass band, with respect to the antenna input resistance at the center frequency $\mathrm{f_0}$.

Due to the small variation in the antenna input resistance within the bandpass band, the antenna input reactance is responsible of the antenna bandwidth and, for this reason, its variation must carefully be taken under control. This input reactance is a function of the top-load type and operation frequency. The lower the frequency, the greater the input reactance variation, and more difficult is to achieve the necessary bandwidth for a broadcast transmission.

For a high fidelity AM transmission, a bandwidth of ± 10 kHz minimum is necessary, and for a VSWR less than 1.25 should be the ideal.

This is a very difficult task to be achieved, especially in the low frequency band. Bandwidth calculations have been carried out for different top-loaded antennas, in both low and medium frequency bands, in order to determine if their behavior is compatible with the previous task. For this reason, bandwidth calculations for VSWR of 1.25, 1.50 and 2.00 have been carried out on each band.

In the low frequency band $(150 - 250$ kHz$)$, the antenna bandwidths calculated at 200 kHz for the Inverted-L, T and X antennas and for different soil conditions can be seen in Figs. 27, 28 and 29.

Clearly, the antenna impedance is very sharp and the VSWR is very high at the specified bandwidth of ± 10 kHz. This problem does not permit a high fidelity transmission, but only speech transmissions. At the same time, it can be seen that the best result is obtained using the X antenna, because an improved bandwidth is obtained compared to the L and T antenna types. Using a top-load with more branches, like an 8-Star antenna, the bandwidth is quite similar to the X antenna, and it does not pay the investment in wiring and support towers.

In the medium frequency band $(535 - 1705$ kHz$)$, the antenna bandwidths calculated at 1 MHz for the Inverted-L and T antennas and for different soil conditions can be seen in Figs. 30 and 31.

From these figures, an improved bandwidth has been achieved compared to the low frequency band behavior of these antennas, nevertheless, for a low VSWR operation the

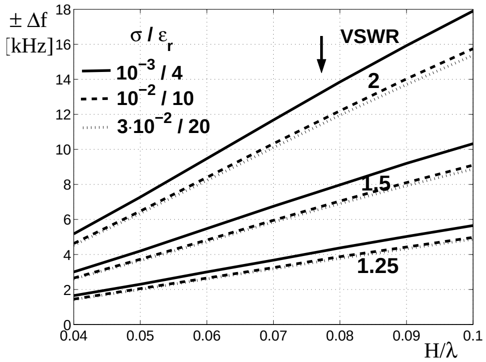

Fig. 31. Resonant T antenna bandwidth as a function of the antenna height and for different soil conditions at 1 MHz. ($\mathrm{n}=2$, $\mathrm{n_c}=1$, $\mathrm{a}=6\cdot10^{-3}$ m, $\mathrm{R_0}=0.25\lambda$, $\mathrm{N}=120$, $\mathrm{a_0}=1.5\cdot10^{-3}$ m).

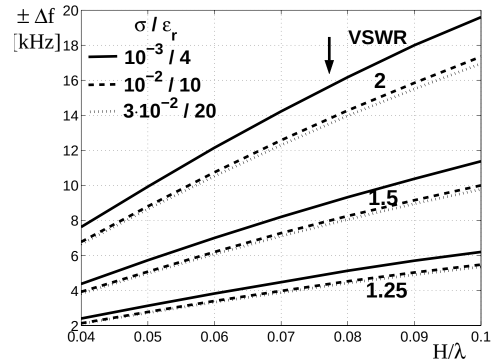

Fig. 33. Resonant X antenna bandwidth as a function of the antenna height and for different soil conditions at 1 MHz. ($\mathrm{n}=4$, $\mathrm{n_c}=1$, $\mathrm{a}=6\cdot10^{-3}$ m, $\mathrm{R_0}=0.25\lambda$, $\mathrm{N}=120$, $\mathrm{a_0}=1.5\cdot10^{-3}$ m).

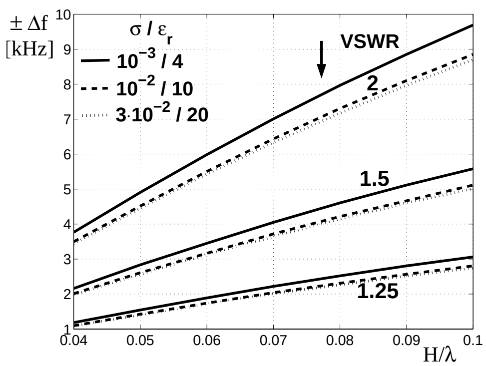

Fig. 32. Resonant X antenna bandwidth as a function of the antenna height and for different soil conditions at 550 kHz. ($\mathrm{n}=4$, $\mathrm{n_c}=1$, $\mathrm{a}=6\cdot10^{-3}$ m, $\mathrm{R_0}=0.25\lambda$, $\mathrm{N}=120$, $\mathrm{a_0}=1.5\cdot10^{-3}$ m).

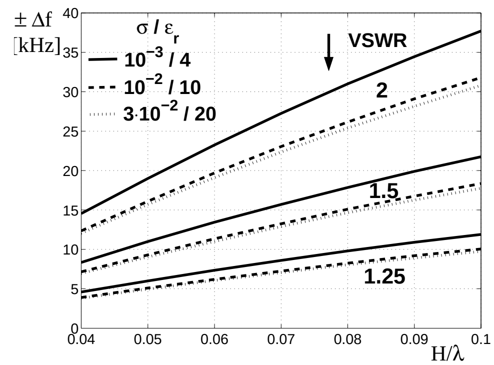

Fig. 34. Resonant X antenna bandwidth as a function of the antenna height and for different soil conditions at 1.7 MHz. ($\mathrm{n}=4$, $\mathrm{n_c}=1$, $\mathrm{a}=6\cdot10^{-3}$ m, $\mathrm{R_0}=0.25\lambda$, $\mathrm{N}=120$, $\mathrm{a_0}=1.5\cdot10^{-3}$ m).

antenna impedance is still quite sharp. For the Inverted-L antenna, it can be seen that a bandwidth of ±5 kHz is achieved for a VSWR of 1.5 and, in the T antenna case, a bandwidth of ±8 kHz is achieved for the same VSWR, for $\mathrm{H}=0.08\lambda$.

From previous calculations, the X antenna has an improved bandwidth compared to both the Inverted-L and T antennas. For this reason, the X antenna bandwidth was calculated, and it is shown in Figs. 32, 33 and 34, for 550 kHz, 1 MHz and 1.7 MHz, respectively.

Clearly, the X antenna has a bandwidth of ±10 kHz for a VSWR of 1.5 at the frequency of 1 MHz, for an antenna height close to 0.08λ. Of course, it is sharper at 550 kHz and it has a wider bandwidth at 1.7 MHz, exceeding the required ±10 kHz, as it can be appreciated in the figures.

According to these results, care must be taken in order to choose these antennas for a high fidelity operation, especially taking into account the frequency within the medium frequency AM band, because, in the lower part, the required ±10 kHz bandwidth is difficult to be obtained for a very low VSWR operation.

This problem can be attenuated choosing a higher antenna, because the antenna bandwidth can be improved increasing the antenna height. This problem can be very difficult to be solved if the antenna height is lower than 0.07λ, especially in the lower part of the medium frequency band. In the upper part, even an antenna height close to 0.05λ can be used with a good bandwidth, but the antenna gain can suffer due to a low radiation resistance, especially for dry soils.

In a low budget case and in the upper part of the band, the Inverted-L and T antennas can be used, because in this part of the band they can offer enough bandwidth for a moderate broadcast operation.

XIII. TOP-LOAD TIP VOLTAGE

The top-load tip voltage $\mathrm{V_L}$ (28) has been calculated for the resonant Inverted-L $(\mathrm{n}=1)$, T $(\mathrm{n}=2)$, X $(\mathrm{n}=4)$ and 8-Star $(\mathrm{n}=8)$ antennas. This knowledge is very important in order to design the supporting insulators, especially when the antenna has to work with high power.

In Figs. 35 and 36 the tip effective voltage can be seen as a function of the antenna height, for an antenna input power of 1 kW and for average soil. The tip voltage calculation requires the knowledge of the antenna equivalent circuit, whose components permit the antenna current determination.

This voltage is very high in the case of the Inverted-L antenna and for the lower antenna heights. At the same time, it can be seen that this voltage is smaller as the branches of the top-load are increasing. When the antenna height is close to 0.1λ, this voltage is quite similar for any type of loading.

For an X antenna, the tip voltage is moderate and almost independent of the antenna height for heights higher than 0.04λ.

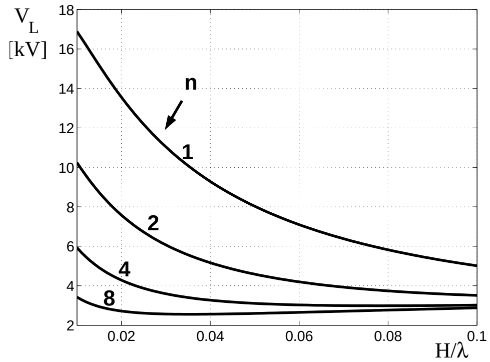

Fig. 35. Top-load tip effective voltage for resonant top-loaded antennas as a function of the antenna height at 200 kHz. ($\mathrm{n_c} = 1$, $\mathrm{a} = 6 \cdot 10^{-3}$ m, $\mathrm{R_0} = 0.05\lambda$, $\mathrm{N} = 30$, $\mathrm{a_0} = 1.5 \cdot 10^{-3}$ m, $\sigma = 10^{-2}$ S/m, $\epsilon_\mathrm{r} = 10$).

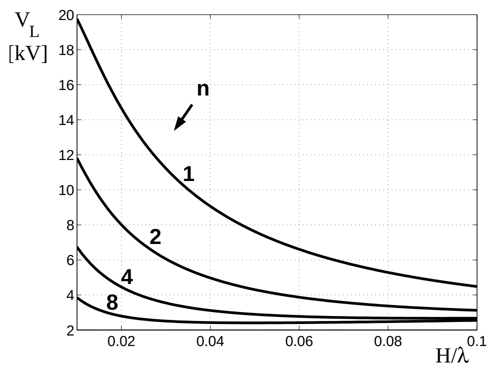

Fig. 36. Top-load tip effective voltage for resonant top-loaded antennas as a function of the antenna height at 1 MHz. ($\mathrm{n_c} = 1$, $\mathrm{a} = 6 \cdot 10^{-3}$ m, $\mathrm{R_0} = 0.25\lambda$, $\mathrm{N} = 120$, $\mathrm{a_0} = 1.5 \cdot 10^{-3}$ m, $\sigma = 10^{-2}$ S/m, $\epsilon_\mathrm{r} = 10$).

XIV. Antenna Wiring

Low and medium frequency top-loaded antennas are made up of metallic wiring and, for achieving the antenna resonance for a given height, the top-load wire length L must be varied to a value of $\mathrm{L_{res}}$ (12).

In Fig. 37 the antenna total length $\mathrm{H_{tot}} = \mathrm{H} + \mathrm{n}\,\mathrm{L_{res}}$ value for the Inverted-L $(\mathrm{n} = 1)$, T $(\mathrm{n} = 2)$, X $(\mathrm{n} = 4)$ and 8-Star $(\mathrm{n} = 8)$ antennas is presented as a function of the antenna height (H/λ) calculated at 1 MHz.

This figure shows that the Inverted-L antenna is using the minimum top-load length $\mathrm{L_{res}}$, compared to the other antenna types. At the same time, it is interesting to see that, approximately, a quarter-wave total wire length $\mathrm{H_{tot}}$ is needed for this antenna to achieve the resonance condition, and this effect occurs for any antenna height. This result is practically the same for any frequency expressing the dimensions in wavelengths, and the small difference is due to the equivalent transmission lines characteristic impedance ratio $\mathrm{Z_{0t}}/\mathrm{Z_{0m}}$, as it is indicated in (12).

This effect is exclusive for the resonant Inverted-L antenna and it does not occur for the other antenna types.

XV. Gain and Field Strength

The antenna community is using the term antenna gain G with a reference to an isotropic source. This term is expressed

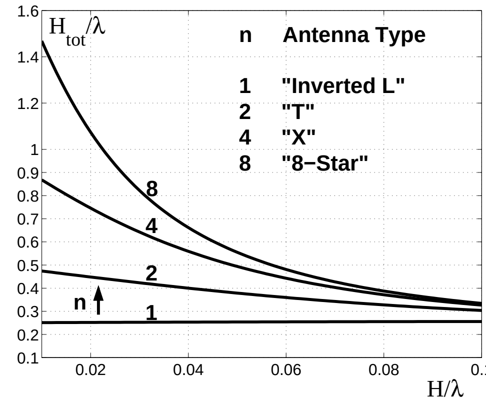

Fig. 37. Resonant top-loaded antenna total length $(\mathrm{H_{tot}} = \mathrm{H} + \mathrm{n}\,\mathrm{L_{res}})$ as a function of the antenna height, for different top-load branches n at 1 MHz.

in times or dBi, and it is independent of the distance. Electric field strength is a magnitude that depends on the distance and, for this reason, a reference distance is of common use.

This reference distance is generally chosen to be 1 km and field strength E is expressed in mV/m or dBμV/m for an antenna input power of 1 kW. Usually, this distance is assumed to be adequate for analyzing a radiated field, free of induction effects. This is almost true for the medium frequency band, where 1 km is a distance close to 1.8 wavelengths at the band lowest frequency.

Nevertheless, a distance of 1 km is half-wavelength at 150 kHz.

In this latter case, this distance should be increased to higher values, at least to 5 km, in order to measure an actual radiating field free of any induction component.

In Table XIII antenna gains are indicated and the unattenuated E-field strength values are presented at the distance of 1 km for an antenna input power of 1 kW .

XVI. Conclusion

Top-loaded antennas have been analyzed. From this analysis, it is pointed out the following:

- Efficiency could be very high when the antenna height is higher than 0.07λ and with an artificial ground plane of 0.25λ and 120 radials in medium frequency band. Field strength will be close to a quarter-wave monopole case. In low frequency band, an optimum artificial ground plane is achieved with a radius of 0.1λ and 30 or 60 radials.
- Radiation resistance is not only a function of the antenna height, but a function of the top-base current relationship too.
- Radiation pattern is found to be a complicated mathematical function of the zenith angle θ. In the case of a very short top-loaded antenna, this function reduces to a simple $\sin\theta$ (see (116) in Appendix C). As a consequence, the directivity is a little bit greater than 4.77 dBi (see Appendix D).
- Inverted-L antenna has been found to have the minimum wiring in order to achieve the self-resonance. It was found that the wire total length $(\mathrm{H_{tot}})$ is independent

TABLE XIII
ANTENNA GAIN AND E-FIELD STRENGTH

G	G	E	E
dBi	—	$\mathrm{mV/m}$	$\mathrm{dB\mu V/m}$
5.0	3.16	307.9	109.76
4.8	3.02	301.0	109.57
4.6	2.88	294.0	109.37
4.4	2.75	287.2	109.16
4.2	2.63	280.9	108.97
4.0	2.51	274.4	108.77
3.8	2.40	268.3	108.57
3.6	2.29	262.1	108.37
3.4	2.19	256.3	108.18
3.2	2.09	250.4	107.97
3.0	2.00	244.9	107.78
2.8	1.90	238.7	107.56
2.6	1.82	233.7	107.37
2.4	1.74	228.5	107.18
2.2	1.66	223.2	106.97
2.0	1.58	217.7	106.76
1.8	1.51	212.8	106.56
1.6	1.45	208.6	106.38
1.4	1.38	203.5	106.17
1.2	1.32	199.0	105.98
1.0	1.26	194.4	105.77

of the antenna height. Unfortunately, this antenna has the minimum bandwidth compared to the other top-loaded antennas, so this effect must be taken into account before choosing the right antenna model.
This antenna can advantageously be used for other services when the bandwidth is not a constraint, due to its simple construction.

- Wire loss resistance depends on the top-load type and on the current distribution along the antenna wiring.
- Separation between radiated and dissipated power is a very difficult task. For these reason, these calculations have a logical limitation, due to the use of a hemispherical surface half-wavelength in radius for the ground plane power dissipation. However, measurements indicate that very good results are obtained from this approach.
- Ground plane equivalent loss resistance is not only a function of the lengths and number of radials and soil constants, but it is a function of the antenna height too.
- Antenna bandwidth is highly dependent on the antenna type and on the operation frequency. An important bandwidth increase can be achieved using an X antenna type, instead of an Inverted-L or T. Increasing the top-load branches more than four, the increase in bandwidth is quite small, and the antenna complexity is going to be very high.
  In the low frequency band, bandwidth is quite scarce for any top-loaded antenna type and must carefully be evaluated in order to obtain a good quality speech transmission. In this band, this kind of antennas is practically the only choice, due to the antenna size.
  In the medium frequency band low end, it is quite difficult to obtain a high fidelity bandwidth. This antenna property improves with the frequency and, in the upper end of this band, even an Inverted-L or T type can give the necessary bandwidth for a broadcast high fidelity transmission.
- Ground plane must carefully be chosen in order to get an optimum performance, and it must be free of obstacles up to a half-wavelength radius for several reasons:
  (a) For an optimum antenna operation.
  (b) For personnel protection, due to the high intensity fields close to the antenna when the input power is higher than $1\ \mathrm{kW}$.
  (c) A short antenna does not mean that it can be installed in the small plot surrounded by obstacles, because its performance can suffer notably and, for this reason, the transmitting house must be installed at a minimum distance of half-wavelength away from the antenna place.
  (d) These conditions must be fulfilled.

APPENDIX A
CURRENT AND VOLTAGE DISTRIBUTIONS

For the antenna vertical wire, it is well known that the current and voltage distributions along a low loss transmission line, of characteristic impedance $\mathrm{Z_{0m}}$, are given by [8]

$$\mathrm{I(z) = I_0\,\cos\beta z - j\,\frac{V_0}{Z_{0m}}\,\sin\beta z} \tag{71}$$

$$\mathrm{V(z) = V_0\,\cos\beta z - j\,I_0\,Z_{0m}\,\sin\beta z} \tag{72}$$

$$\mathrm{0 \le z \le H}$$

Where $\mathrm{I_0}$ and $\mathrm{V_0}$ are the antenna input current and voltage.

$$\mathrm{V_0 = j\,X_a\,I_0} \tag{73}$$

$$\mathrm{I(z) = I_0\left(\cos\beta z + \frac{X_a}{Z_{0m}}\,\sin\beta z\right)} \tag{74}$$

$$\mathrm{V(z) = j\,I_0\,(X_a\,\cos\beta z - Z_{0m}\,\sin\beta z)} \tag{75}$$

$$\mathrm{0 \le z \le H}$$

At the antenna top, $\mathrm{z = H}$,

$$\mathrm{\frac{I_t}{I_0} = \frac{I(z=H)}{I_0} = \cos\beta H + \frac{X_a}{Z_{0m}}\,\sin\beta H} \tag{76}$$

$$\mathrm{V_t = V(z=H) = j\,I_0\,(X_a\,\cos\beta H - Z_{0m}\,\sin\beta H)} \tag{77}$$

The current and voltage distributions on the top-load, considered as a low loss transmission line, of characteristic impedance $\mathrm{Z_{0t}}$, will be [8]

$$\mathrm{I(\rho) = \frac{I_t}{n}\,\cos\beta\rho - j\,\frac{V_t}{Z_{0t}}\,\sin\beta\rho} \tag{78}$$

$$V(\rho) = V_t \, \cos\beta\rho - j\,\frac{I_t}{n}\,Z_{0t}\,\sin\beta\rho \tag{79}$$

$$0 \le \rho \le L$$

Where I_t and V_t are the antenna top current and voltage.

$$V_t = j\,X_t\,I_t \tag{80}$$

Taking into account (5), it follows that

$$I(\rho) = \frac{I_t}{n}\left(\cos\beta\rho - \frac{\sin\beta\rho}{\tan\beta L}\right) \tag{81}$$

$$V(\rho) = j\,I_t\left(X_t\,\cos\beta\rho - \frac{Z_{0t}}{n}\,\sin\beta\rho\right) \tag{82}$$

$$0 \le \rho \le L$$

At the top-load wire tip,

$$I_L = I(\rho = L) = 0 \tag{83}$$

$$V_L = V(\rho = L) = j\,I_t\left(X_t\,\cos\beta L - \frac{Z_{0t}}{n}\,\sin\beta L\right) \tag{84}$$

If the antenna is resonant, $X_a = 0$, then

$$I(z) = I_0\,\cos\beta z \tag{85}$$

$$V(z) = -j\,I_0\,Z_{0m}\,\sin\beta z \tag{86}$$

$$0 \le z \le H$$

$$\frac{I_t}{I_0} = \cos\beta H \tag{87}$$

$$V_t = -j\,I_0\,Z_{0m}\,\sin\beta H \tag{88}$$

$$I(\rho) = \frac{I_t}{n}\left(\cos\beta\rho - \frac{\sin\beta\rho}{\tan\beta L_{res}}\right) \tag{89}$$

$$V(\rho) = -j\,I_0\,\cos\beta H\left(Z_{0m}\,\tan\beta H\,\cos\beta\rho + \frac{Z_{0t}}{n}\,\sin\beta\rho\right) \tag{90}$$

$$0 \le \rho \le L_{res}$$

$$I_L = 0 \tag{91}$$

$$V_L = -j\,I_0\,\frac{Z_{0t}\,\cos\beta H}{n\,\sin\beta L_{res}} \tag{92}$$

APPENDIX B
NEAR FIELD

Near fields will be calculated in cylindrical coordinates, according to Fig. 4. Therefore,

$$R^2 = (z - z')^2 + \rho^2 \tag{93}$$

$$r_1^2 = (z - H)^2 + \rho^2 \tag{94}$$

$$r_2^2 = (z + H)^2 + \rho^2 \tag{95}$$

$$r^2 = z^2 + \rho^2 \tag{96}$$

The magnetic vector potential in free space, according to the current distribution along the z-axis, has only one component in the z-direction, that is

$$A_z = \frac{\mu_0}{4\,\pi}\int_{-H}^{H} I(z')\,\frac{e^{-j\beta R}}{R}\,dz' \tag{97}$$

Making a variable change, the current distribution in (74) becomes

$$I(z) = \begin{cases} I_m\,\sin(\psi^+ + \beta z) & \text{for } -H \le z \le 0 \\ -I_m\,\sin(\psi^- - \beta z) & \text{for } \quad 0 \le z \le H \end{cases} \tag{98}$$

Where

$$I_m = I_0\sqrt{1 + \left(\frac{X_a}{Z_{0m}}\right)^2} \tag{99}$$

$$\psi = \arctan\left(\frac{X_a}{Z_{0m}}\right) \tag{100}$$

$$\psi^{\pm} = \psi \pm \frac{\pi}{2} \tag{101}$$

Thus,

$$\begin{aligned} A_z = \frac{\mu_0\,I_m}{4\,\pi}\Bigg[\int_{-H}^{0} \sin(\psi^+ + \beta z')\,\frac{e^{-j\beta R}}{R}\,dz' \\ - \int_{0}^{H} \sin(\psi^- - \beta z')\,\frac{e^{-j\beta R}}{R}\,dz'\Bigg] \end{aligned} \tag{102}$$

or

$$\begin{aligned} A_z = j\,\frac{\mu_0\,I_m}{8\,\pi}\Bigg[e^{j\psi^-}\int_{0}^{H} \frac{e^{-j\beta(R+z')}}{R}\,dz' \\ - e^{-j\psi^-}\int_{0}^{H} \frac{e^{-j\beta(R-z')}}{R}\,dz' \\ - e^{j\psi^+}\int_{-H}^{0} \frac{e^{-j\beta(R-z')}}{R}\,dz' \\ + e^{-j\psi^+}\int_{-H}^{0} \frac{e^{-j\beta(R+z')}}{R}\,dz'\Bigg] \end{aligned} \tag{103}$$

Then, the magnetic and electric fields are given by

$$\mathrm{H}_\phi = -\frac{1}{\mu_0}\,\frac{\partial \mathrm{A_z}}{\partial \rho} \tag{104}$$

$$\mathrm{E_z} = -\frac{\mathrm{j}}{\omega\,\epsilon_0}\,\frac{1}{\rho}\,\frac{\partial(\rho\,\mathrm{H}_\phi)}{\partial \rho} \tag{105}$$

Following the same procedure as Jordan [4] and for $\mathrm{z} = 0$, it follows that

$$\begin{aligned}\mathrm{H}_\phi = -\frac{\mathrm{I_m}}{4\,\pi\,\rho}\Bigg\{&\mathrm{e}^{\mathrm{j}\psi}\left[\left(1-\frac{\mathrm{H}}{\mathrm{r}_1}\right)\mathrm{e}^{-\mathrm{j}\beta(\mathrm{r}_1+\mathrm{H})} - \mathrm{e}^{-\mathrm{j}\beta\rho}\right] \\ &- \mathrm{e}^{-\mathrm{j}\psi}\left[\left(1+\frac{\mathrm{H}}{\mathrm{r}_1}\right)\mathrm{e}^{-\mathrm{j}\beta(\mathrm{r}_1-\mathrm{H})} - \mathrm{e}^{-\mathrm{j}\beta\rho}\right]\Bigg\}\end{aligned} \tag{106}$$

$$\begin{aligned}\mathrm{E_z} = \frac{\mathrm{j}\,\mathrm{I_m}}{4\,\pi\,\epsilon_0\,\omega\,\rho}\Bigg\{&\mathrm{e}^{\mathrm{j}\psi}\Bigg[\frac{\rho\,\mathrm{e}^{-\mathrm{j}\beta(\mathrm{r}_1+\mathrm{H})}}{\mathrm{r}_1}\left(\frac{\mathrm{H}}{\mathrm{r}_1^2} - \mathrm{j}\beta\Big(1-\frac{\mathrm{H}}{\mathrm{r}_1}\Big)\right) \\ &+\mathrm{j}\beta\,\mathrm{e}^{-\mathrm{j}\beta\rho}\Bigg] \\ &- \mathrm{e}^{-\mathrm{j}\psi}\Bigg[-\frac{\rho\,\mathrm{e}^{-\mathrm{j}\beta(\mathrm{r}_1-\mathrm{H})}}{\mathrm{r}_1}\left(\frac{\mathrm{H}}{\mathrm{r}_1^2} + \mathrm{j}\beta\Big(1+\frac{\mathrm{H}}{\mathrm{r}_1}\Big)\right) \\ &+\mathrm{j}\beta\,\mathrm{e}^{-\mathrm{j}\beta\rho}\Bigg]\Bigg\}\end{aligned} \tag{107}$$

At resonance, $\mathrm{X_a} = 0$, so $\mathrm{I_m} = \mathrm{I_0}$ and $\psi = 0$, then the magnetic and electric fields, for $\mathrm{z} = 0$, will be

$$\mathrm{H}_\phi = \frac{\mathrm{I_0}}{2\,\pi}\,\frac{\mathrm{e}^{-\mathrm{j}\beta\mathrm{r}_1}}{\rho}\,\left(\frac{\mathrm{H}}{\mathrm{r}_1}\,\cos\beta\mathrm{H} + \mathrm{j}\,\sin\beta\mathrm{H}\right) \tag{108}$$

$$\mathrm{E_z} = \frac{\mathrm{j}\,\mathrm{I_0}\,\mathrm{e}^{-\mathrm{j}\beta\mathrm{r}_1}}{2\,\pi\,\epsilon_0\,\omega}\left(\frac{\mathrm{H}\,\cos\beta\mathrm{H}}{\mathrm{r}_1^3} + \frac{\mathrm{j}\,\beta\mathrm{H}\,\cos\beta\mathrm{H}}{\mathrm{r}_1^2} - \frac{\beta\,\sin\beta\mathrm{H}}{\mathrm{r}_1}\right) \tag{109}$$

The magnetic field expression is exactly the same obtained by Wait and Surtees [11] by means of a different approach, assuming a sinusoidal antenna current distribution. In this presentation, the current distribution has been obtained by means of an equivalent transmission line model. In this case, the maximum current $\mathrm{I_m}$ in (99) depends on the antenna reactance $\mathrm{X_a}$ and is going to be $\mathrm{I_0}$ when the antenna is resonant ($\mathrm{X_a} = 0$), while the ψ parameter in (100) is going to be zero.

Appendix C
Far Field

The far fields can be obtained from (104) and (105) using the transformation from cylindrical to spherical coordinates. Therefore,

$$\begin{cases}\rho = \mathrm{r}\,\sin\theta \\ \phi = \phi \\ \mathrm{z} = \mathrm{r}\,\cos\theta\end{cases} \tag{110}$$

Also, the following approximations can be applied:

$$\mathrm{r}_1 \cong \mathrm{r} - \mathrm{H}\,\cos\theta \tag{111}$$

$$\mathrm{r}_2 \cong \mathrm{r} + \mathrm{H}\,\cos\theta \tag{112}$$

Then, the far magnetic and electric fields, in the upper hemisphere ($0 \le \theta \le \pi/2$), are given by

$$\mathrm{H}_\phi = \mathrm{j}\,\frac{\mathrm{I_m}}{2\,\pi}\,\frac{\mathrm{e}^{-\mathrm{j}\beta\mathrm{r}}}{\mathrm{r}}\,\mathrm{f}_\psi(\theta) \tag{113}$$

$$\mathrm{E}_\theta = \mathrm{j}\,\mathrm{Z}_{00}\,\frac{\mathrm{I_m}}{2\,\pi}\,\frac{\mathrm{e}^{-\mathrm{j}\beta\mathrm{r}}}{\mathrm{r}}\,\mathrm{f}_\psi(\theta) \tag{114}$$

Where

$$\begin{aligned}\mathrm{f}_\psi(\theta) = &\frac{\sin(\beta\mathrm{H}-\psi)\cos(\beta\mathrm{H}\cos\theta)}{\sin\theta} \\ &+\frac{\sin\psi - \cos(\beta\mathrm{H}-\psi)\cos\theta\sin(\beta\mathrm{H}\cos\theta)}{\sin\theta}\end{aligned} \tag{115}$$

is **the top-loaded monopole antenna field radiation pattern**, $\mathrm{Z}_{00} = 377\ \Omega$ is the free space intrinsic impedance, $\mathrm{I_m}$ is given by (99) and ψ by (100).

At resonance, $\mathrm{X_a} = 0$, so $\mathrm{I_m} = \mathrm{I_0}$ and $\psi = 0$, then **the resonant top-loaded monopole antenna field radiation pattern** is given by

$$\mathrm{f}_0(\theta) = \frac{\sin\beta\mathrm{H}\cos(\beta\mathrm{H}\cos\theta) - \cos\beta\mathrm{H}\cos\theta\sin(\beta\mathrm{H}\cos\theta)}{\sin\theta} \tag{116}$$

In the case of a very short top-loaded antenna, $\beta\mathrm{H} \ll 1$ in (116), then the far magnetic and electric fields are exactly the same of the Hertz monopole. Therefore,

$$\mathrm{H}_\phi = \mathrm{j}\,\frac{\beta\,\mathrm{H}\,\mathrm{I_0}}{2\,\pi}\,\frac{\mathrm{e}^{-\mathrm{j}\beta\mathrm{r}}}{\mathrm{r}}\,\sin\theta \tag{117}$$

$$\mathrm{E}_\theta = \mathrm{j}\,\mathrm{Z}_{00}\,\frac{\beta\,\mathrm{H}\,\mathrm{I_0}}{2\,\pi}\,\frac{\mathrm{e}^{-\mathrm{j}\beta\mathrm{r}}}{\mathrm{r}}\,\sin\theta \tag{118}$$

Where

$$\mathrm{f}(\theta) = \sin\theta \tag{119}$$

is **the Hertz monopole field radiation pattern**.

If $\theta = \pi/2$, then $\mathrm{r} = \rho$, $\mathrm{z} = 0$ and $\mathrm{f}_0(\pi/2) = \sin\beta\mathrm{H}$. Thus, the far magnetic and electric fields on the earth surface, for any resonant top-loaded antenna, become

$$\mathrm{H}_\phi = \mathrm{j}\,\frac{\mathrm{I_0}}{2\,\pi}\,\frac{\mathrm{e}^{-\mathrm{j}\beta\rho}}{\rho}\,\sin\beta\mathrm{H} \tag{120}$$

$$\mathrm{E}_\theta = \mathrm{j}\,\mathrm{Z}_{00}\,\frac{\mathrm{I_0}}{2\,\pi}\,\frac{\mathrm{e}^{-\mathrm{j}\beta\rho}}{\rho}\,\sin\beta\mathrm{H} \tag{121}$$

Since $\mathrm{E_z} = -\mathrm{E}_\theta$ for $\mathrm{z} = 0$, it follows that the z-component of the far electric field on the earth surface becomes

$$E_z = -j\, Z_{00}\, \frac{I_0}{2\,\pi}\, \frac{e^{-j\beta\rho}}{\rho}\, \sin\beta H \tag{122}$$

APPENDIX D
DIRECTIVITY

The antenna directivity D can be expressed as

$$D = \frac{4\,\pi}{B} \tag{123}$$

Where

$$B = \int_0^{2\pi} d\phi \int_0^{\pi/2} \frac{P(\theta,\phi)}{P_{max}} \sin\theta\, d\theta \tag{124}$$

is the beam area, $P(\theta,\phi)$ is the power density and P_{max} its maximum value. When the antenna beam has cylindrical symmetry, the power density is only a function of the zenith angle θ, so $P(\theta,\phi) = P(\theta)$.

For a resonant top-loaded antenna,

$$\frac{P(\theta)}{P_{max}} = \frac{f_0^2(\theta)}{f_0^2(\pi/2)} \tag{125}$$

Where $f_0(\theta)$ is the resonant top-loaded antenna field radiation pattern in (116).

The integration in (124) can be carried out to give

$$B = \frac{\pi}{2\,\sin^2\beta H}\left[\frac{\sin 4\beta H}{4\beta H} + \frac{\sin 2\beta H}{2\beta H} - \cos 2\beta H - 1 + \mathrm{Cin}(4\beta H)\right] \tag{126}$$

Where

$$\mathrm{Cin}(4\beta H) = \int_0^{4\beta H} \frac{1-\cos u}{u}\, du \tag{127}$$

In Table XIV the resonant top-loaded antenna directivity has been calculated as a function of the antenna height (H/λ). Exact results are very close to the Hertz monopole directivity of 4.77 dBi within 0.1 dB.

TABLE XIV
RESONANT TOP-LOADED ANTENNA DIRECTIVITY.

H/λ	D	D
—	—	dBi
0.010	3.0008	4.7724
0.015	3.0018	4.7738
0.020	3.0032	4.7758
0.025	3.0049	4.7783
0.030	3.0071	4.7814
0.035	3.0096	4.7851
0.040	3.0125	4.7893
0.045	3.0158	4.7941
0.050	3.0195	4.7993
0.055	3.0235	4.8051
0.060	3.0279	4.8114
0.065	3.0327	4.8182
0.070	3.0377	4.8255
0.075	3.0432	4.8332
0.080	3.0489	4.8414
0.085	3.0549	4.8500
0.090	3.0613	4.8591
0.095	3.0680	4.8685
0.100	3.0749	4.8783

APPENDIX E
RADIATION RESISTANCE

Radiation resistance is defined as [5]

$$R_{rad} = \frac{2\,W_{rad}}{I_0^2} \tag{128}$$

Where W_{rad} is the power radiated into space by the top-loaded antenna, and I_0 is the peak value of the antenna input current.

Following the standard procedure [5], the radiation resistance of a resonant top-loaded antenna will be

$$R_{rad} = 60 \int_0^{\pi/2} f_0^2(\theta)\, \sin\theta\, d\theta \tag{129}$$

Where $f_0(\theta)$ is the resonant top-loaded antenna field radiation pattern in (116).

The integration can be performed analytically to give

$$R_{rad} = 15\left[\frac{\sin 4\beta H}{4\beta H} + \frac{\sin 2\beta H}{2\beta H} - \cos 2\beta H - 1 + \mathrm{Cin}(4\beta H)\right] \tag{130}$$

Where the Cin function is given by (127).

In Table XV the resonant top-loaded antenna radiation resistance has been calculated using the exact expression (130), the approximate equation (54), where the top to base current ratio $I_t/I_0 = \cos\beta H$ is taken into account, and the Hertz monopole radiation resistance (44), where $I_t/I_0 = 1$.

It can be seen that the Hertz monopole expression (44) can only be used for antenna heights less than 0.04λ, while the approximate expression (54) is valid up to 0.1λ with an error less than 5%.

APPENDIX F
WIRE LOSS RESISTANCE

It was shown in Section VIII that the wire loss resistance for a resonant antenna is given by

$$R_c = \frac{R_l}{I_0^2}\left(\int_0^H I^2(z)\, dz + n \int_0^{L_{res}} I^2(\rho)\, d\rho\right) \tag{131}$$

Where

$$R_l = \frac{1}{a}\sqrt{\frac{f\,\mu_0}{4\,\pi\,\sigma_c}} \tag{132}$$

and

TABLE XV
RESONANT TOP-LOADED ANTENNA RADIATION RESISTANCE.

H/λ	Exact	Approx.	Hertz
—	Ω	Ω	Ω
0.010	0.15766	0.15760	0.15791
0.015	0.35405	0.35373	0.35531
0.020	0.62768	0.62668	0.63165
0.025	0.97727	0.97485	0.98696
0.030	1.40120	1.39620	1.42120
0.035	1.89740	1.88810	1.93440
0.040	2.46360	2.44790	2.52660
0.045	3.09710	3.07200	3.19780
0.050	3.79500	3.75700	3.94780
0.055	4.55400	4.49870	4.77690
0.060	5.37070	5.29270	5.68490
0.065	6.24110	6.13440	6.67190
0.070	7.16140	7.01890	7.73780
0.075	8.12740	7.94090	8.88260
0.080	9.13460	8.89500	10.1060
0.085	10.1780	9.87570	11.4090
0.090	11.2540	10.8770	12.7910
0.095	12.3580	11.8940	14.2520
0.100	13.4830	12.9190	15.7910

$$\mathrm{I(z)} = \mathrm{I_0}\,\cos\beta\mathrm{z} \qquad 0 \le \mathrm{z} \le \mathrm{H} \tag{133}$$

$$\mathrm{I}(\rho) = \frac{\mathrm{I_0}\,\cos\beta\mathrm{H}}{\mathrm{n}}\left(\cos\beta\rho - \frac{\sin\beta\rho}{\tan\beta\mathrm{L_{res}}}\right) \tag{134}$$

$$0 \le \rho \le \mathrm{L_{res}}$$

Both integrations in (131) can be carried out to give

$$\int_0^{\mathrm{H}} \mathrm{I}^2(\mathrm{z})\,\mathrm{dz} = \frac{\mathrm{I}_0^2}{2}\left(\mathrm{H} + \frac{\sin 2\beta\mathrm{H}}{2\,\beta}\right) \tag{135}$$

and

$$\int_0^{\mathrm{H}} \mathrm{I}^2(\rho)\,\mathrm{d}\rho = \frac{\mathrm{I}_0^2\,\cos^2\beta\mathrm{H}}{\mathrm{n}^2}\left[\frac{\mathrm{L_{res}}}{2}\left(1 + \frac{1}{\tan^2\beta\mathrm{L_{res}}}\right)\right. \tag{136}$$
$$\left. + \frac{\sin 2\beta\mathrm{L_{res}}}{4\,\beta}\left(1 - \frac{1}{\tan^2\beta\mathrm{L_{res}}}\right) + \frac{\cos 2\beta\mathrm{L_{res}} - 1}{2\,\beta\,\tan\beta\mathrm{L_{res}}}\right]$$

Therefore,

$$\mathrm{R_c} = \mathrm{R_l}\left\{\frac{1}{2}\left(\mathrm{H} + \frac{\sin 2\beta\mathrm{H}}{2\,\beta}\right)\right. \tag{137}$$
$$+ \frac{\cos^2\beta\mathrm{H}}{\mathrm{n}}\left[\frac{\mathrm{L_{res}}}{2}\left(1 + \frac{1}{\tan^2\beta\mathrm{L_{res}}}\right)\right.$$
$$\left.\left. + \frac{\sin 2\beta\mathrm{L_{res}}}{4\,\beta}\left(1 - \frac{1}{\tan^2\beta\mathrm{L_{res}}}\right) + \frac{\cos 2\beta\mathrm{L_{res}} - 1}{2\,\beta\,\tan\beta\mathrm{L_{res}}}\right]\right\}$$

APPENDIX G
GROUND PLANE EQUIVALENT LOSS RESISTANCE

In Section IX, the ground plane equivalent loss resistance was obtained as

$$\mathrm{R_{gp}} = \frac{2\pi}{\mathrm{I}_0^2}\left(\int_0^{\mathrm{R_0}} |\,\mathrm{H}_\phi\,|^2\ \mathrm{R_g}\,\rho\,\mathrm{d}\rho + \int_{\mathrm{R_0}}^{\lambda/2} |\,\mathrm{H}_\phi\,|^2\ \mathrm{R_s}\,\rho\,\mathrm{d}\rho\right) \tag{138}$$

Where

H_ϕ is the near magnetic field given by (108) $[\mathrm{A/m}]$.

$\mathrm{R_g}$ is the artificial ground plane resistance at the operation frequency, given by (62) $[\Omega]$.

$\mathrm{R_s}$ is the soil resistance at the operation frequency, given by (60) $[\Omega]$.

The first integral is calculated numerically, thus

$$\frac{2\pi}{\mathrm{I}_0^2}\int_0^{\mathrm{R_0}} |\,\mathrm{H}_\phi\,|^2\ \mathrm{R_g}\,\rho\,\mathrm{d}\rho \cong \frac{2\pi}{\mathrm{I}_0^2}\sum_{\mathrm{i=1}}^{\mathrm{i=K}} |\,\mathrm{H}_\phi(\rho_\mathrm{i})\,|^2\ \mathrm{R_g}(\rho_\mathrm{i})\,\rho_\mathrm{i}\,\mathrm{w_i} \tag{139}$$

Where $\{\mathrm{w_i}\}$ are the weights of an adaptive Gauss-Lobatto quadrature rule.

The second integration can be carried out to give

$$\frac{2\pi}{\mathrm{I}_0^2}\int_{\mathrm{R_0}}^{\lambda/2} |\,\mathrm{H}_\phi\,|^2\ \mathrm{R_s}\,\rho\,\mathrm{d}\rho = \tag{140}$$
$$\frac{\mathrm{R_s}}{2\,\pi}\left\{\ln\left(\frac{\lambda}{\mathrm{R_0}}\sqrt{\frac{\mathrm{R}_0^2 + \mathrm{H}^2}{\lambda^2 + 4\,\mathrm{H}^2}}\right)\cos^2\beta\mathrm{H}\right.$$
$$\left. + \ln\left(\frac{\lambda}{2\,\mathrm{R_0}}\right)\sin^2\beta\mathrm{H}\right\}$$

APPENDIX H
GLOSSARY OF SYMBOLS

a	Radius of the antenna wires [m].
A	Magnetic vector potential $[\mathrm{Wb/m}]$.
$\mathrm{a_0}$	Radius of wire used in artificial ground plane [m].
B	Radiation pattern beam area.
$\mathrm{C_t}$	Top capacitance of a top-loaded antenna [F].
D	Antenna directivity.
η	Antenna efficiency.
E	Electric field intensity [V/m].
ϵ_r	Soil relative permittivity.
$\mathrm{f_0}(\theta)$	Resonant top-loaded antenna field radiation pattern.
$\mathrm{f}_\psi(\theta)$	Top-loaded antenna field radiation pattern.
G	Antenna gain.
H	Magnetic field intensity [A/m].
H	Antenna height [m].
$\mathrm{I_0}$	Peak value of the antenna input current [A].
$\mathrm{I}(\rho)$	Current distribution on the antenna top-load [A].
$\mathrm{I_t}$	Peak value of the antenna top current [A].
$\mathrm{I(z)}$	Current distribution on the antenna vertical part [A].
j	$\sqrt{-1}$ imaginary unit.

$\mathbf{J}_{\mathrm{su}}$ Ground plane surface current density [A/m].
L Antenna top-load length [m].
$\mathrm{L}_{\mathrm{res}}$ Resonant antenna top-load length [m].
n Number of top-load branches.
n_{c} Number of wires in each equivalent transmission line.
N Number of radials.
$\mathbf{P}$ Radiated power density $[\mathrm{W/m^2}]$.
Q Antenna merit factor.
ρ Radial distance from the antenna base [m].
R_0 Artificial ground plane radius [m].
R_{a} Antenna input resistance $[\Omega]$.
R_{c} Wire loss resistance $[\Omega]$.
R_{g} Artificial ground plane resistance $[\Omega]$.
R_{gp} Ground plane equivalent loss resistance $[\Omega]$.
R_{i} Insulator equivalent loss resistance $[\Omega]$.
R_{l} Wire resistance per unit length $[\Omega/\mathrm{m}]$.
$\mathrm{R}_{\mathrm{rad}}$ Antenna radiation resistance $[\Omega]$.
R_{s} Soil resistance $[\Omega]$.
σ Soil conductivity [S/m].
σ_{c} Wire conductivity [S/m].
V_0 Peak value of the antenna input voltage [V].
V_{L} Peak value of the antenna top-load tip voltage [V].
$\mathrm{V}(\rho)$ Voltage distribution on the antenna top-load [V].
V_{t} Peak value of the antenna top voltage [V].
$\mathrm{V}(\mathrm{z})$ Voltage distribution on the antenna vertical part [V].
W_{c} Power dissipated in the antenna wires [W].
W_{d} Power dissipated in the antenna ground plane [W].
W_{in} Antenna input power [W].
$\mathrm{W}_{\mathrm{rad}}$ Antenna radiated power [W].
X_{a} Antenna input reactance $[\Omega]$.
X_{t} Antenna top reactance $[\Omega]$.
Z_0 Near field space impedance $[\Omega]$.
Z_{00} Free space intrinsic impedance $(377\ \Omega)$.
$\mathrm{Z}_{0\mathrm{m}}$ Equivalent transmission line average characteristic impedance of the antenna vertical part $[\Omega]$.
$\mathrm{Z}_{0\mathrm{t}}$ Equivalent transmission line characteristic impedance of the antenna top-load $[\Omega]$.
Z_{a} Antenna input impedance $[\Omega]$.
Z_{g} Artificial ground plane impedance $[\Omega]$.
Z_{r} Ground screen impedance $[\Omega]$.
Z_{s} Soil impedance $[\Omega]$.
Z_{t} Antenna top impedance $[\Omega]$.

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