Dual-Z0 Shortened Stubs

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Introduction

Shorted and open-circuit stubs, located along a transmission line, have long been a method of achieving a match or notch through the impedance transformations, the inductive reactance, or the capacitive reactance that stubs provide. Such stub techniques are discussed in The ARRL Handbook and The ARRL Antenna Book. (A detailed discussion of stub applications and stub matching techniques is beyond the scope of this article.) What is presented here is a method of shortening those matching stubs by as much as 66% by taking advantage of the phase shift between two pieces of transmission line with differing Z0s (characteristic impedances).

The topic that seems to have been ignored in amateur radio publications is the physical length advantage to be gained by using dual-Z0 shortened stubs with two differring Z0 characteristic impedances. Since the pure reactance circle on a Smith Chart is associated with a phase angle and the normalized value of reactance is a tangent function of that phase angle, we can use the ARCTAN function to determine the phase shift in degrees at an impedance discontinuity in a stub. (This lossless phase shift at an impedance discontinuity ***point*** in a stub seems to be a free lunch granted to us mere mortals by the transmission line gods.) Assuming a reactance of 0-j300 ohms and normalizing to Z01 = 600 ohms and Z02 = 50 ohms:

ARCTAN(300/600) = 26.6 deg = 0.0739 wavelength

ARCTAN(300/50) = 80.5 deg = 0.2236 wavelength

If the impedance is 0+j300 ohms at an impedance discontinuity between Z01 = 600 ohm line and Z02 = 50 ohm line, a steady-state phase shift of 80.5 - 26.6 = 53.9 deg will occur at that point when the impedance is purely reactive, i.e. an infinite SWR exists. The maximum possible shortening of a stub occurs at the highest Z0H/Z0L ratio and at the shorter physical lengths. (Z0H is the high-Z0 section and Z0L is the low-Z0 section.) For instance, when Z0H/Z0L = 12/1, one can obtain a 30 degree electrical length from an ideal stub that is only 5 degrees long physically, i.e. 1/6 of the physical length of a single-Z0 stub.

For the purpose of this ***theoretical*** paper, all transmission lines and stubs will be considered to be ideal and lossless. The velocity factor, VF, of the feedlines is assumed to be 1.0 but will obviously require consideration in an actual real-world implementation when the VF is not 1.0. This paper applies only to stubs used at the single frequency of interest. Unlike single-Z0 stubs, a 1/4 wavelength dual-Z0 shortened stub does not act like a 1/2 wavelength stub at double the frequency of interest. It can be seen from the graph in Fig. 4 that the electrical length to physical length function is not a straight line when two different Z0s are used.

It is unlikely that the information contained in this paper is original but the author has been unable to locate any references in amateur radio publications that explain and present information about dual-Z0 shortened stubs. However, what transpires at an impedance discontinuity in a transmission line has been covered in a previous article. [1] This paper is an extension of the principles covered in that earlier article.

The electrical length of a stub varies with frequency. Rather than using a particular frequency for the examples, all examples have been normalized to degrees. They could just have easily been normalized to radians or wavelength. Here are the relationships between frequency, degrees, radians, velocity factor, and wavelength.

Free space wavelength in feet = 984/frequency in MHz

Transmission line wavelength in feet = VF*984/frequency in MHz

Where VF is the velocity factor of the transmission line, e.g. 0.66 for RG-213.

One wavelength = 360 degrees = 2 pi radians

Most amateur radio operators have heard of stubs and stub matching techniques. Unfortunately, useful lengths of stubs may not be feasible on the longer wavelength HF bands, e.g. where 1/4 wavelength is ~65 feet on 3.8 MHz. This paper will present information about a method of shortening stubs by using two lengths of transmission lines of differing characteristic impedances, Z0H and Z0L. The above ~65 feet on 3.8 MHz can be shrunk to ~25 feet using Z0H = 600 ohms and Z0L = 72 ohms transmission line stock (assuming VF = 0.9).

Dual-Z0 Stubs

The relative locations of the two sections of feedline are important. The following figure indicates the correct locations for the high-Z0 line and the low-Z0 line for open stubs and shorted stubs. (Stubs with other than open or shorted terminations are beyond the scope of this paper.)

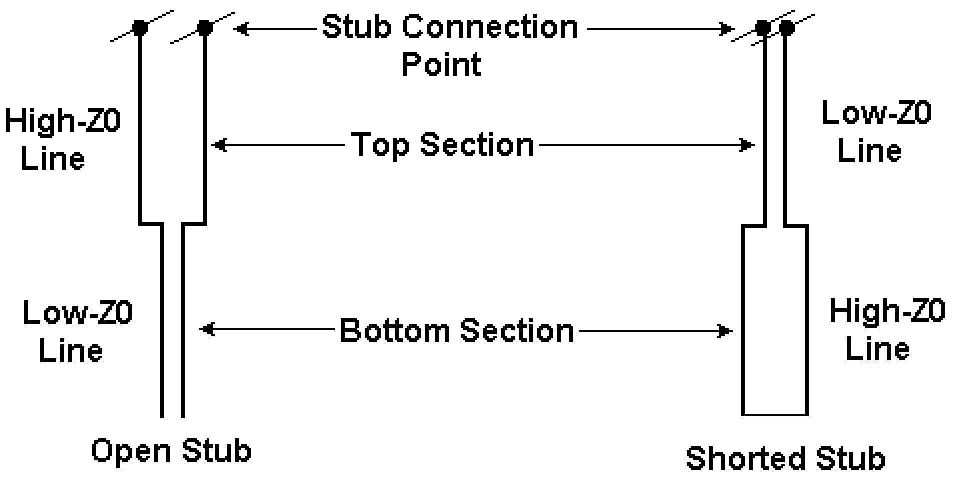


Fig. 1 - Configurations for Shortened Stubs

For open stubs, the low-Z0 section <u>must</u> be placed at the open (bottom) unconnected end of the stub. The high-Z0 section becomes the top (connected) end.

For shorted stubs, the high-Z0 section <u>must</u> be placed at the shorted (bottom) unconnected end of the stub. The low-Z0 section becomes the top (connected) end.

Let's assume that we have two types of transmission lines available with differing characteristic impedances, Z0H ohms and Z0L ohms (where Z0H > Z0L) and we want to make a physically shortened stub that is electrically longer than the two physical lengths combined. It appears that the most efficient result occurs when the two section lengths are equal in degrees. If the VFs of both sections are equal, the two sections will also be equal in physical length. It turns out that the actual values of the characteristic impedances are not important. It is the ratio of Z0H/Z0L that is important. The frequency is also not important as everything is expressed in degrees which can be converted to wavelength by dividing by 360. The equation that governs such physically shortened stubs is:

$$\mathbf{TELD = TPLD/2 + ARCTAN\,[(Z0H/Z0L)TAN(TPLD/2)]} \qquad \textbf{Eq. 1}$$

Where: TELD is the Total Electrical Length in Degrees of the Dual-Z0 stub. TPLD is the Total Physical Length in Degrees of the stub and assumes that the two different Z0 sections are equal in number of physical degrees. Z0H/Z0L is the ratio of Z0H to Z0L.

As an example, let's assume that we create an open stub made from Z0H = 600 ohm feedline and Z0L = 72 ohm feedline such that Z0H/Z0L = 8.3333. Assume that each section is physically 19.11 degrees long making the total stub 38.22 degrees long physically. What will be the electrical length in degrees?

$$\mathbf{TELD = 38.22/2 + ARCTAN[(8.33)TAN(38.22/2)] = 90\ degrees = 1/4\ wavelength}$$

Need a 1/4 wavelength open stub that is 43% of the length required by a normal 1/4 wavelength open stub? We have created that electrically long 1/4 wavelength stub using only 19.11 + 19.11 = 38.22 physical degrees of transmission line in a dual-Z0 configuration. The secret is that the impedance discontinuity provides a 51.87 deg phase shift in the middle of the stub as illustrated in the following graphic.

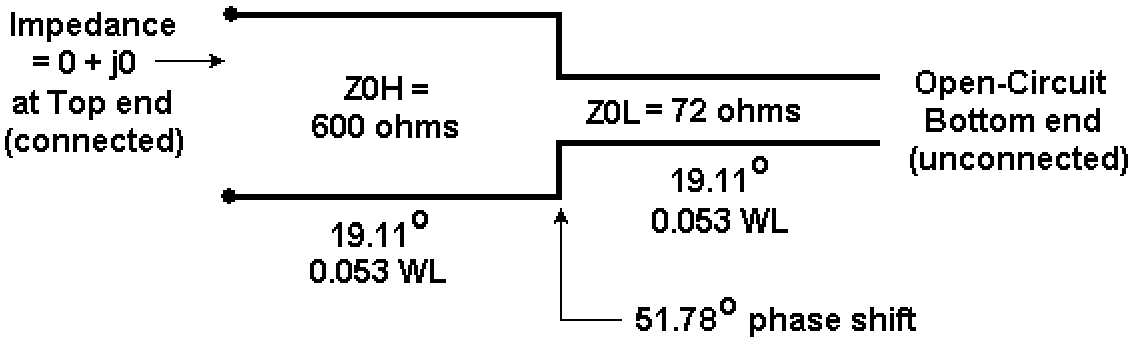
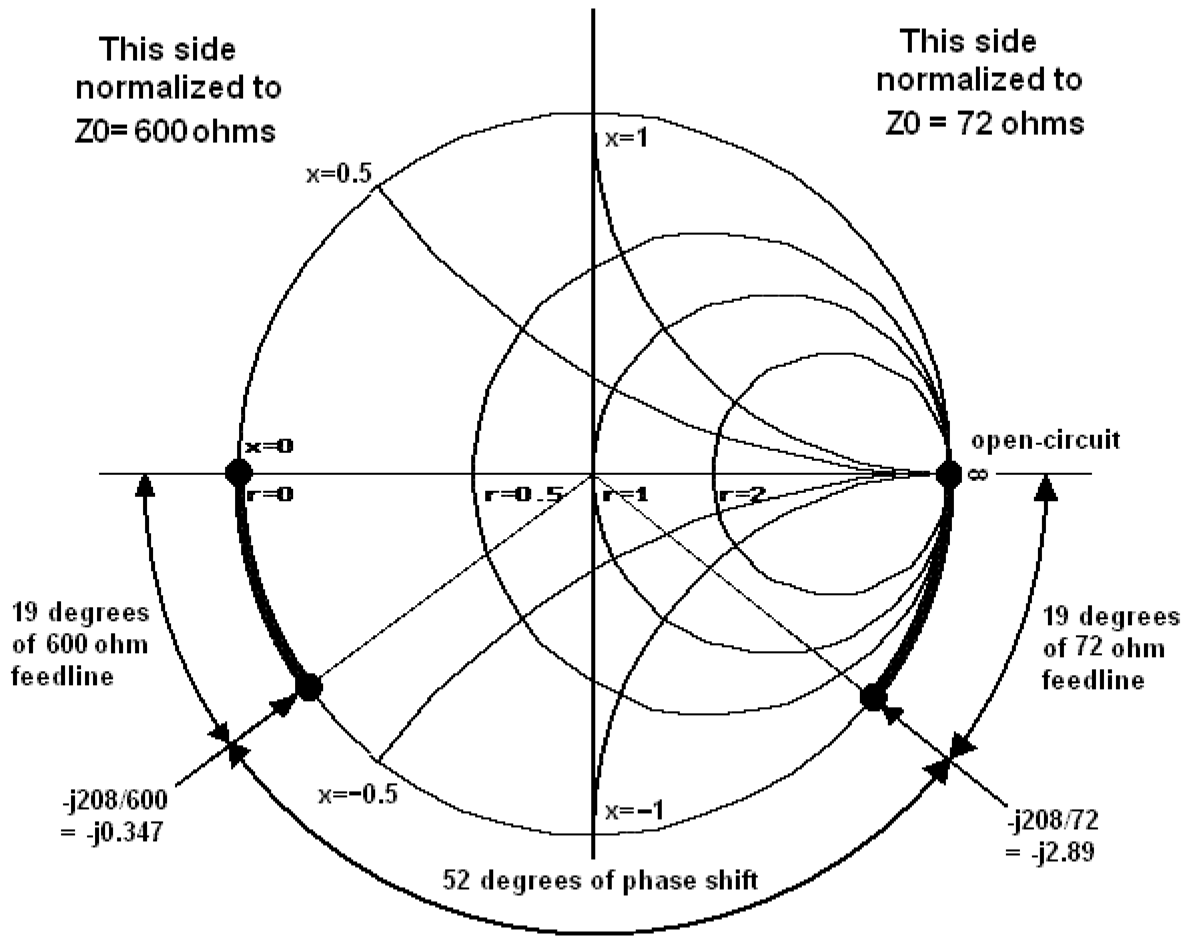


Fig. 2 - A Physically Short 1/4 WL Open-Circuit Stub

For those who can read a Smith Chart, here is a graphical representation of this stub.

Shortened 1/4WL Open-Circuit Stub



Note how the same impedance, -j208, gets shifted by the appropriate phase shift when normalized to either 72 ohms or 600 ohms.

If we take the identical stub above and swap ends such that the Z0H becomes the shorted bottom end of a shorted stub and the Z0L end becomes the top connected end, the stub becomes an electrical 1/4 wavelength shorted stub. (If the ends are not swapped when going from open-stub to shorted-stub, the desired shortening effect of the dual-Z0 stub will not be realized.) See Fig. 1 above.

The stub can also have other electrical lengths besides 90 degrees. What electrical length would we get from a shorted-stub with the following characteristics?

TPLD = 25 degrees (Total physical degrees assuming equal length sections)

Z0H/Z0L = 6 (Z0H = 300 and Z0L = 50)

TELD = 25/2 + ARCTAN[(6.0)TAN(25/2)] = 65.56 degrees = 0.182 wavelength

The physical length is 25 degrees. The electrical length is 65.56 degrees. The dual-Z0 stub is 38% of the length of a single-Z0 stub because of the 40.56 degree phase shift at the impedance discontinuity.

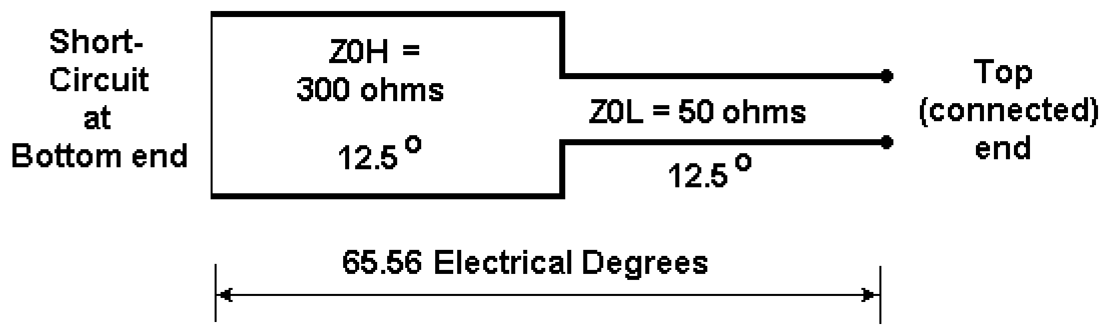


Fig. 3 - A Physically Short Shorted Stub

1/4 Wavelength Stubs

For the special case of 1/4 wavelength stubs, the following equation applies:

TPLD = 2{ARCTAN[SQRT(Z0L/Z0H)]}   Eq. 2

If Z0H = 600 ohms and Z0L = 72 ohms, each of the two sections is 19.11 degrees which is the example given above in Fig. 2. Use the above equation for creating shortened 1/4 wavelength stubs.

One interesting combination of characteristic impedances would be Z0 = 600 ohm open-wire line combined with a side-by-side parallel run of 50 ohm coax which has a Z0 of 100 ohms. With the Z0H/Z0L ratio of 600/100, a 1/4 wavelength dual-Z0 stub will be almost exactly one half the length of the single Z0 stub.

Graphical Solution

The following graph allows one to choose the total electrical length required for a stub using a particular Z0H/Z0L ratio to determine the total physical length in degrees for the shortened stub using equal lengths (degrees) of sections of Z0H and Z0L feedlines.

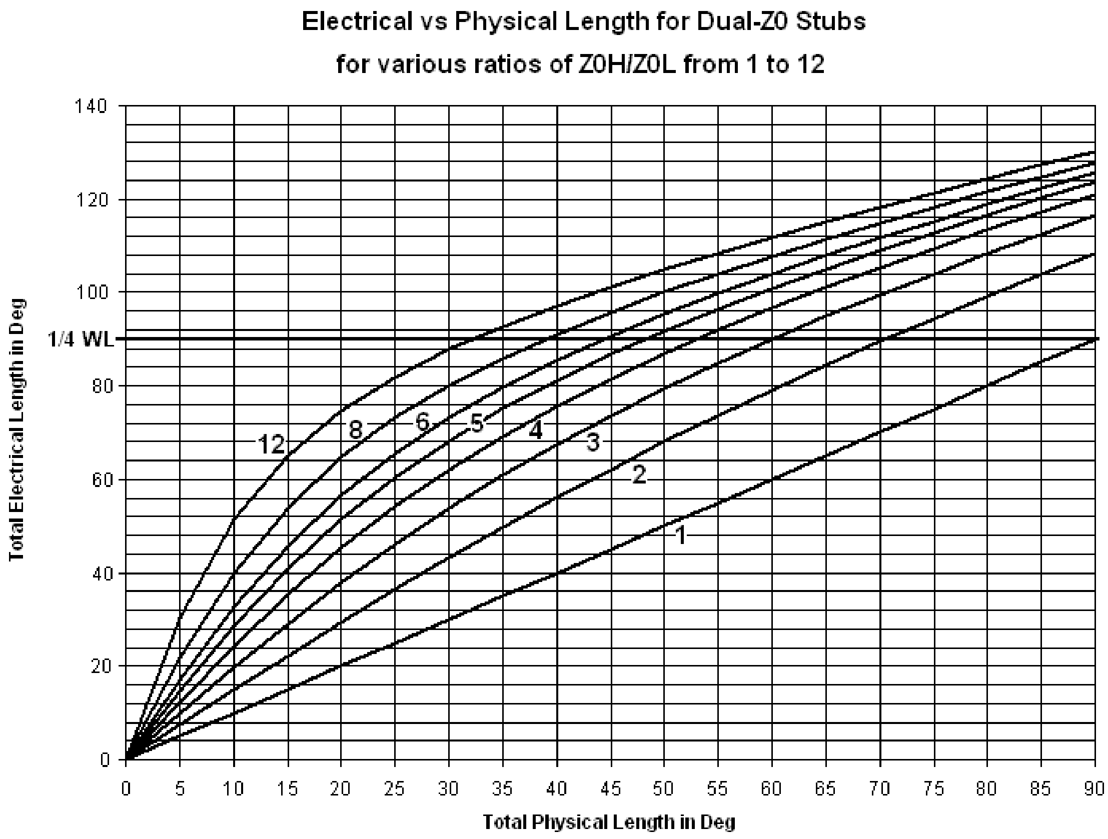


Fig. 4 - Electrical Vs Physical Lengths of Stubs

It should be obvious that the above curves converge where the total electrical length and the total physical length are both equal to 180 degrees, i.e. 1/2 wavelength. At that convergence point, The Z0s of the two sections doesn't matter at all as long as they are equal.

1/2 Wavelength Transmission Lines

The above technique can also be used to shorten transmission lines. For instance, assume one has a 75m dipole with a feedpoint impedance of 50+j0 ohms at resonance. If we want to match that 50 ohms by feeding the dipole with 1/2 wavelength of Z0 = 600 ohm feedline, we will need 116.5 feet on 3.8 MHz assuming a velocity factor of 0.9. Can we shorten the feedline? Of course we can by using a piece of Z0 = 100 ohm feedline in the middle of a Z0 = 600 ohm feedline section. (The Z0 = 100 ohm feedline can be two side-by-side runs of 50 ohm coax.)

Using the above graph, we find that the physical length of a shortened 1/4 wavelength stub when the Z0H/Z0L ratio is 6:1 is approximately 45 degrees. So if we make each end of the feedline equal to ~22.5 degrees of Z0 = 600 ohm feedline and make the middle section equal to ~45 degrees of Z0 = 100 ohm feedline, then we will have created a 1/2 wavelength (180 deg) shortened feedline that is physically approximately 1/4 wavelength (90 deg) long.

If the Z0 = 600 ohm feedline has a VF of 0.9, the two lengths will be close to 14.6 feet each. If the Z0 = 100 ohm feedline has a VF of 0.66, the center length will be close to 21.4 feet. Thus we have created an electrically 180 degree long feedline using only about 36 feet of feedline. Our shortened electrical 1/2 wavelength feedline is only about 31% of the length of the original 1/2 wavelength of Z0 = 600 ohm feedline.

Note that the Z0H = 600 ohm line is connected to the low-Z, 50 ohm antenna. This is in accordance with the requirements in Fig. 1, above.

References

[1] "An Energy Analysis at an Impedance Discontinuity in an RF Transmission Line", by Cecil Moore, W5DXP, Worldradio, Oct 2005 - Jan 2006. Also available at: Energy Analysis

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